IEA DHC|CHP

TWO-STEP DECISION AND OPTIMISATION MODEL FOR CENTRALISED OR DECENTRALISED THERMAL STORAGE IN DH&C SYSTEMS

IEA DISTRICT HEATING AND COOLING, ANNEX VII:

Report 8DHC-05.02
**TWO-STEP DECISION AND OPTIMISATION MODEL FOR
CENTRALISED OR DECENTRALISED THERMAL
STORAGE IN DH&C SYSTEMS**

Executive Summary

Main objectives

The objective of this project has been to develop a decision and optimisation methodology for optimum dimensioning of centralised or decentralised thermal storage in DH&C systems.

Work

This report presents a methodology for assisting the planning of introducing thermal storage into a DH&C plant. The methodology is divided into the solving of two sub problems; the existence problem, in this report referred to as the step one problem and the dimensioning problem referred to as the step two problem.

Chapter 3 contains a study on the technical design of the store and how the shape of the store affects the efficiency.

Chapter 4 comprises a survey on short term operational optimisation of DH&C plants as well as a discussion on the storage optimisation problem and how optimisation uncertainties affect the dimensioning.

The methodology for solving the existence problem, i.e. to find out whether a store should be further investigated or not, is presented in chapter 5. The methodology is based on historical data and the main idea behind this decision model is to study periods where the heat load is fluctuating around the maximum capacity of the base heat production. The necessity and appropriate size of a store is evaluated by calculating the demand of energy above and below the actual production limit.

Solving the dimensioning problem is to find the optimum size of heat storage for a given district heating plant. A methodology for solving this problem is presented in chapter 6. Non-linear operational optimisation models from the literature survey are used to determine the optimal operation of the system (dispatch problem) and dynamic programming is employed for finding the optimal size of the storage (unit commitment).

Conclusions

Minimising the heat loss to the surroundings of a cylinder shaped storage implies an H/D ratio equal to 1,0. However, if the most important issue is to minimise the amount of useless volume the best H/D ratio is 2,0. However some additional height is inevitable due to the space required for diffusors and, if present, also for steam pockets. This is quite consistent with the existing storages in the Nordic countries where a majority of the stores have a H/D ratio range of 1,0 – 2,0.

A simplified method for calculating the economic benefits by using heat storage in different district heating systems is presented and demonstrated. The method is based on historical data. The calculations for some cases show that the method is applicable as a first approach for an investment decision in heat storage.

A methodology for solving the storage dimensioning problem is presented together with a numeric example with a district heat load from a real DH system and where the heat is produced by a back pressure steam turbine CHP in combination with a oil fired heat boiler. For an annual heat load of 712 000 MWh the optimal heat storage volume is approximately 27 000 m^3 and the savings are about 3 % of the annual running costs.

General Preface Annex VII

Introduction

The International Energy Agency (IEA) was established in 1974 in order to strengthen the co-operation between member countries and reduce the dependency on oil and other fossil fuels. Thirty years later, the IEA again drew attention to serious concerns about energy security, investment, the environment and energy poverty. The global situation is resulting in soaring oil and gas prices, the increasing vulnerability of energy supply routes and ever-increasing emissions of climate-destabilising carbon dioxide.

The IEA's *World Energy Outlook*[1] "Reference Scenario" 2004 projects that, in the absence of new government policies or accelerated deployment of new technologies, world primary energy demand will rise by 59% by 2030, with 85% of that increase from the use of coal, oil and natural gas. However, these trends are not unalterable. The *World Energy Outlook* "Alternative Policy Scenario" shows that more vigorous government action and accelerated deployment of new technologies could steer the world onto a markedly different energy path, where world energy demand would be 10% lower and carbon-dioxide emissions 16% lower.

DHC makes a difference

One of the key technologies that can make a difference is District Heating and Cooling. DHC is an integrative technology that **can** make significant contributions to reducing emissions of carbon dioxide and air pollution and to increasing energy security.

The fundamental idea of DHC is simple but powerful: connect multiple thermal energy users through a piping network to environmentally optimum energy sources, such as combined heat and power (CHP), industrial waste heat and renewable energy sources such as biomass, geothermal and natural sources of heating **and** cooling.
The ability to assemble and connect thermal loads enables these environmentally optimum sources to be used in a cost-effective way, and also offers ongoing fuel flexibility. By integrating district cooling carbon-intensive electrically-based airconditioning, rapidly growing in many countries, can be displaced.

As an element of the International Energy Agency Programme, the participating countries undertake co-operative actions in energy research, development and demonstration.

One of the programmes that has run for more than 25 years is the Implementing Agreement 'District Heating and Cooling including the integration of Combined Heat and Power'.

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[1] The annual *World Energy Outlook* presents long-term projections for supply and demand of oil, gas, coal, renewable energy sources, nuclear power and electricity. It also assesses energy-related carbon dioxide emissions and policies designed to reduce them. The annual World Energy Outlook has long been recognized as the authoritative source for global long-term energy market analysis. This flagship publication from the IEA is produced by the agency's Economic Analysis Division with input from other internal and external energy experts as required. For more information see http://www.worldenergyoutlook.org/.

Annex VII
In May 2002 Annex VII started.

Following is a list of the recent research projects (annexes) undertaken by the District Heating & Cooling Implementing Agreement. Ten countries participated from Europe, North America and Asia: Canada, Denmark, Finland, Germany, Korea, The Netherlands, Norway, Sweden, United Kingdom, United States.

Project title	Company	
A comparison of distributed CHP/DH with large-scale CHP/DH	Parsons Brinckerhoff Ltd Formerly PB Power Ltd – Energy **Project leader: Paul Woods**	8DHC-05.01
Two-step decision and optimisation model for centralised or decentralised thermal storage in DH&C	SP Swedish National Testing and Research Institute **Project Leader: John Rune Nielsen**	8DHC-05.02
Improvement of operational temperature differences in district heating systems	ZW Energiteknik **Project leader: Heimo Zinko**	8DHC-05.03
How cellular gases influence insulation properties of district heating pipes and the competitiveness of district energy	Danish Technological Institute **Project leader: Henning D. Smidt**	8DHC-05.04
Biofouling and microbiologically influenced corrosion in district heating networks	Danish Technological Institute **Project Leader: Bo Højris Olesen**	8DHC-05.05
Dynamic heat storage optimization and Demand Side Management	Fraunhofer Institut Umwelt-, Sicherheits-, Energietechnik UMSICHT **Projectleader: Michael Wigbels**	8DHC-05.06
Strategies to manage heat losses – Technique and Economy	MVV Energie AG Technology and Innovationsmanagement **Project leader: Frieder Schmitt**	8DHC-05.07

Benefits of membership
Membership of this implementing agreement fosters sharing of knowledge and current best practice from many countries including those where:
- DHC is already a mature industry
- DHC is well established but refurbishment is a key issue
- DHC is not well established.

Membership proves invaluable in enhancing the quality of support given under national programmes. Participant countries benefit through the active participation in the programme of their own consultants and research organisations. Each of the projects is supported by a team of experts, one from each participant country. As well as the final research reports, other benefits include the cross-fertilisation of ideas which has resulted not only in shared knowledge but also opportunities for further collaboration.

New member countries are very welcome – please simply contact us (see below) to discuss.

Information
General information about the IEA Programme District Heating and Cooling, including the integration of CHP can be obtained from our website www.iea-dhc.org or from:

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Nomenclature

Parameter

a	Investment instalment factor: $a = \dfrac{i}{1-(1+i)^{-N}}$	[€]
A_{surf}	Store surface area	[m^2]
β	Coefficient of volumetric expansivity	[K^{-1}]
C_b	Peak unit annual cost	[€]
c_b	Heat boiler cost	[€]
$c_{b,i}$	Peak unit specific investment cost	[€/W]
c_c	Back pressure CHP cost	[€]
c_{dp}	Accumulated transition cost, dynamic programming	[€]
c_{dp}^{*}	Smallest accumulated transition cost, dynamic programming	[€]
c_e	Electricity price	[€/Wh]
$c_{e\,\lim}$	Minimum electricity price variation limit	[€/Wh]
c_{f_b}	Fuel cost heat boiler	[€/Wh]
c_{f_c}	Fuel cost back pressure CHP	[€/Wh]
c_{f_e}	Fuel cost extraction plant CHP	[€/Wh]
c_g	Engine running cost	[€]
c_{HP}	Heat pump running cost	[€]
c_i	Storage annual cost due to investment	[€]
c_m	Power to heat ratio	[-]
c_{m0}	Slope of condensing pressure line of an extraction plant when no steam is extracted	[-]
c_s	Heat store running cost	[€]
C_s	Heat store annual cost	[€]
$c_{s,ch}$	Heat store charge energy cost	[€/Wh]
$c_{s,i}$	Storage specific investment cost	[€/Wh]
c_{start}	Production unit startup cost	[€]
$c_{start\,x}$	Production unit startup cost coefficients	[€]
c_s^{*}	Current marginal cost	[€/Wh]
c_v	Slope of equi-fuel-line for extraction turbines	[W]
ΔQ_s	Heat store discharge discretisation	[W]
ΔE_s	Heat store energy content discretisation	[Wh]
d_{grad}	Thickness of the storage gradient zone	[m]
d_{wall}	Thickness of the storage wall zone	[m]
D	Store diameter	[m]
$e_{b\,x}$	Electricity usage coefficient, heat boiler	
$e_{c\,x}$	Electricity usage coefficient, back pressure CHP	
$e_{e\,x}$	Electricity usage coefficient, extraction plant CHP	

E_s	Heat store energy content	[Wh]
$E_{s\,max}$	Maximum heat store energy content	[Wh]
E_{tot}	Annually delivered heat	[Wh]
f_b	Fuel consumption, heat boiler	[W]
f_{bx}	Fuel consumption coefficient, heat boiler	
f_c	Fuel consumption back pressure CHP	[W]
f_{cx}	Fuel consumption coefficient, back pressure CHP	
f_e	Fuel consumption extraction plant CHP	[W]
f_{ex}	Fuel consumption coefficient, extraction plant CHP	
f_g	Fuel consumption diesel/gas engine CHP	[W]
f_{g_x}	Fuel consumption coefficient, diesel/gas engine CHP	
γ_1	Heat store heat loss coefficient	[h^{-1}]
γ_2	Heat store pump cost coefficient	[W^{-1}]
γ_3	Heat store charge capacity coefficient	[h^{-1}]
γ_4	Heat store heat capacity coefficient	[Wh/m^3]
γ_5	Heat store heat loss coefficient	[(hm^2)$^{-1}$]
g	Gravity (9.81m/s^2)	m/s^2
h	Gradient zone height	[m]
H	Store height	[m]
i	Interest rate	[%]
I	Store purchase price	[€]
k_0	Constant in the function of the condensing pressure line for an extraction power plant	[W]
λ_{bp}	Back pressure coefficient	[-]
L	Characteristic length	[m]
$LMTD_{cond}$	Logarithmic mean temperature difference heat pump condenser	[°C]
$LMTD_{cond,ref}$	Logarithmic mean temperature difference heat pump condenser at reference state	[°C]
$LMTD_{evap}$	Logarithmic mean temperature difference heat pump evaporator	[°C]
η_{acc}	Store efficiency	[-]
η_b	Boiler efficiency	[-]
η_{carnot}	Heat pump carnot efficiency	[-]
η_{comp}	Heat pump compressor efficiency	[-]
η_e	Generator efficiency	[-]
η_{tot}	Total efficiency	[-]
N	Instalment period	[-]
N_s	Number of annual storage cycles	[-]
P	Power output	[W]
P_d	Power demand	[W]
P_0	Total heat production in a cogeneration plant	[W]
$P_{help,x}$	Electricity usage for unit x	[W]
P_{HP}	Heat pump condenser maximum output capacity	[W]
$P_{HP,ref}$	Heat pump condenser maximum output capacity at reference state	[W]

P_{max}	Power production of a condensing steam turbine in a cogeneration plant at zero heat output	[W]
Q_0	Heat transfer in low pressure condenser for steam extraction power plants	[W]
Q_b	Heat production in a heat boiler	[W]
$Q_{b\,max}$	Maximum heat production in a heat boiler	[W]
Q_c	Heat output from a heat production unit	[W]
$Q_{c\,max}$	Maximum heat production in a back pressure CHP	[W]
Q_d	District heat load	[W]
Q_H	Heat production in a back pressure CHP	[W]
Q_i	Heat load period i	[W]
$\overline{Q_L}$	Average heat load	[W]
$Q_{L\,lim}$	Minimum energy limit	[W]
Q_{prod}	Production limit	[W]
Q_s	Heat discharged from the heat store	[W]
$Q_{s,j}$	Storage size for period j	[W]
$Q_{s\,max}$	Maximum heat store discharge	[W]
Ri	Richardson's number	[-]
r	Store radius	[m]
τ	Store utilization time	[h]
t	Time	[h]
T	Sample period	[h]
T_{cond}	Temperature of condensation, heat pump	[°C]
T_{evap}	Temperature of evaporation, heat pump	[°C]
$T_{f,cond}$	Heat pump condenser forward temperature, DH fluid outlet temperature	[°C]
T_{ret}	DH return temperature, heat pump condenser inlet temperature	[°C]
$T_{source,in}$	Heat pump evaporator heat source fluid inlet temperature	[°C]
$T_{source,out}$	Heat pump evaporator heat source fluid outlet temperature	[°C]
T_{sup}	Supply temperature	[°C]
T_{out}	Outdoor temperature	[°C]
ΔT	Temperature difference between hot and cold side of the store	[°C]
UA_{cond}	Heat pump condenser overall heat transfer coefficient	[W/°C]
v	Velocity	[m/s]
V	Heat store volume	[m³]
V_{grad}	Volume of the storage gradient zone	[m³]
V_{use}	Useful heat storage volume	[m³]
V_{wall}	Volume of the storage wall zone	[m³]
x_k	Dynamic programming state	[-]

Abbreviations

CCOP   Carnot coefficient of performance
CHP   Combined Heat and Power
COP   Coefficient of performance
DH&C   District Heating and Cooling
LMTD   Logarithmic mean temperature difference

1 Objective

Optimisation of district heating or cooling plants is often complex. A high number of variables and constraints imply the need for efficient tools for predesign purposes. Every specific solution of the plant layout gives an exclusive total cost, emission level or any similar measure. In this case the existence and optimum size of a store is considered.

The objective of the project is to develop a decision model and an optimisation methodology for optimum dimensioning of a centralised or decentralised thermal storage in DH&C systems.

This means methodologies for techno-economic analysis and optimisation of DH&C plants.

The decision model shall be a simplified model for supporting a first decision whether thermal storage should be further considered or not. The model shall utilize historical data for the plant i.e. data for heat load, capacities and production prices, to analyse the benefit of a thermal storage by comparing actual operation cost with the cost of running the system with a thermal storage. The difference in these costs can be considered to be the profit from the fictitious thermal storage.

The dimensioning methodology shall be based on existing optimisation models for operational cost in order to find the optimum storage capacity for a given district heat plant. The methodology shall be a combination of a mathematical model and an analytic methodology to handle the uncertainties in the energy price and heat load boundaries of the problem.

2 Introduction to the problem

2.1 Optimisation

Computer aided optimisation within the field of district heating has been practiced since the early 1980s. Although the overall objective has been to find the most economically suitable way to satisfy a district heat demand, various models, optimisation methods and viewing perspectives have been suggested.

The optimisation problem may be categorised by the time horizon considered. For the planning of plant capacity investment as for the planning of heat district augmentation an appropriate time horizon may cover several years whereas the problem of planning the heat production has a much narrower perspective, e.g. days or even hours. In the short term production planning problem, models describing the heat producing units are used for constructing an overall cost function. Minimising the cost function will give the optimal production pattern, i.e. how to engage the available production units. The choice of minimisation method is a matter of complexity and speed.

The problem adressed in this report is the existence and dimensioning problem of a store connected either directly to the production plant or decentralised in the heat district. Questions to be answered are: Should a store be considered? If so, what size? Shape? How much money, energy or emissions will be saved? Should the thermal store be centralised or de-centralised?

To solve the existence problem, in this report referred to as the step one model, is to find out whether a store should be considered further or not. A methodology supporting the decision-making process is presented in chapter 5.

Once it is clear that a store is likely to benefit the production the next step is to estimate the appropriate storage dimensions i.e. given the actual heat production facilities and the energy demand, what is the optimal storage size? A methodology for finding the optimal size of a given plant is presented in chapter 6 and referred to as the step two model.
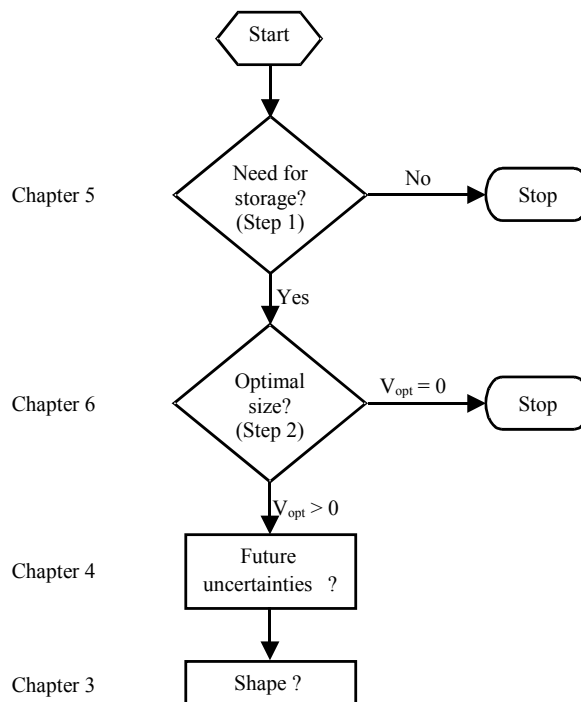

Figure 2-1. The two step decision and optimisation model procedure

2.2 Rational decisions

There are several reasons for installing centralised or decentralised thermal stores in district heating or cooling systems.

These may be:
- To eliminate bottlenecks in production
- To eliminate bottlenecks within the distribution network
- To reduce peak load unit operation
- To improve utilisation of waste heat or base load units
- To enable time shift of heat versus electricity utilisation in case of CHP
- To enhance revenues by taking advantage of time variable energy prices
- To replace the normal heat production during shorter planned or unplanned production stops

The main priority is normally to produce and distribute the energy in the most cost efficient way.

2.3 Pre optimisation analysis

Before starting the decision process of introducing a store, the control of the DH&C system should be evaluated such that load variations originating from badly tuned control systems are eliminated rather than smoothed by a storage. [Werner 1997] found that poor control can be responsible for rather large load variations.

2.4 Limitation of the work

The work does not consider seasonal storage. The methodology considers stores with a cylindrical form, typically steel stores, even though other shapes and functions can be analysed if the model heat loss functions are applicable. The methodology applies to pressurised as well as atmospheric stores.

The work does not consider storage in the district heating or cooling distribution pipes.

The work only considers DH&C systems formed by serial pipes and branch pipes, i.e. DH&C systems containing loops or other thermal shortcuts are not considered.

3 General on heat stores

3.1   General

Hot water can be stored in several types of stores, in this case only steel stores (or stores of equivalent design and functionality) are considered. The work presented in this chapter is to a large extent based on a work report written by Vidar Sundberg [Sundberg 2003].

3.1.1   Existing stores in the nordic countries

Nordvarme [Nordvarme 1993] investigated more than 100 stores in the Nordic countries in 1993, from the smallest at 40m^3 to largest at 96 000m^3. See Figure 3-1 for information on distribution of size, most storages are smaller than 2 500m^3.
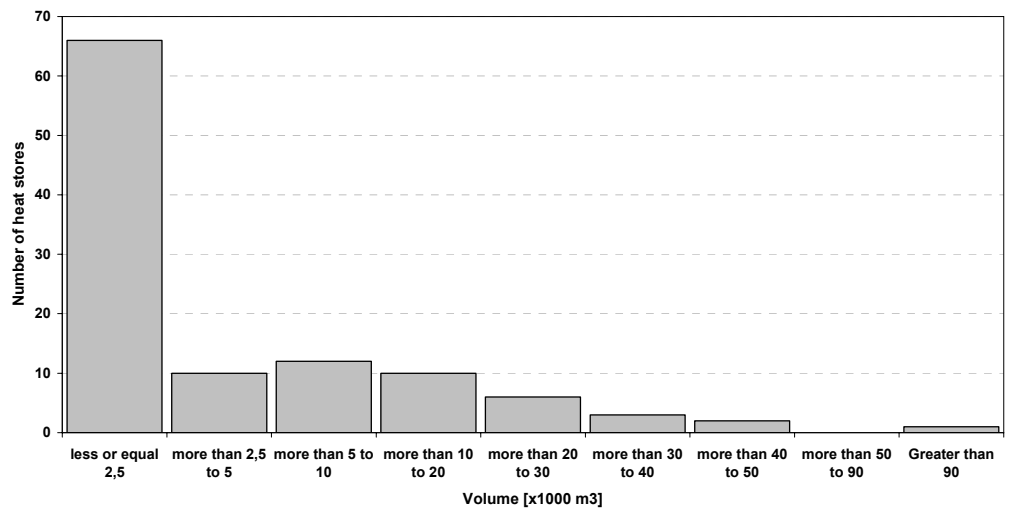

Figure 3-1   Distribution of size.

The storage height versus diameter relation varies a lot, H/D ratios ranging from less than 0,5 to ratios greater than 2,5 as seen in Figure 3-2, however, approximately 50% lie between 1,0 and 2,0.
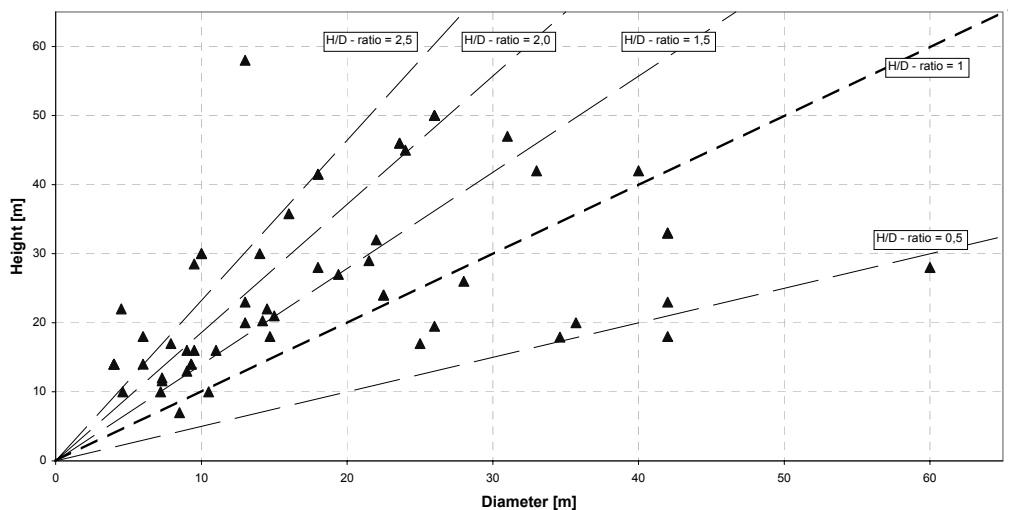

Figure 3-2   Height vs. diameter, and H/D-ratio.

In Figure 3-3 the installed storage volume for the plants covered by [Nordvärme 1993] is related to the annual heat production. For a majority of the plants the installed storage is approximately 50 m^3 per 1000 MWh delivered heat.

Figure 3-3 axis labels:
Storage volume [1000*m3] (y-axis)
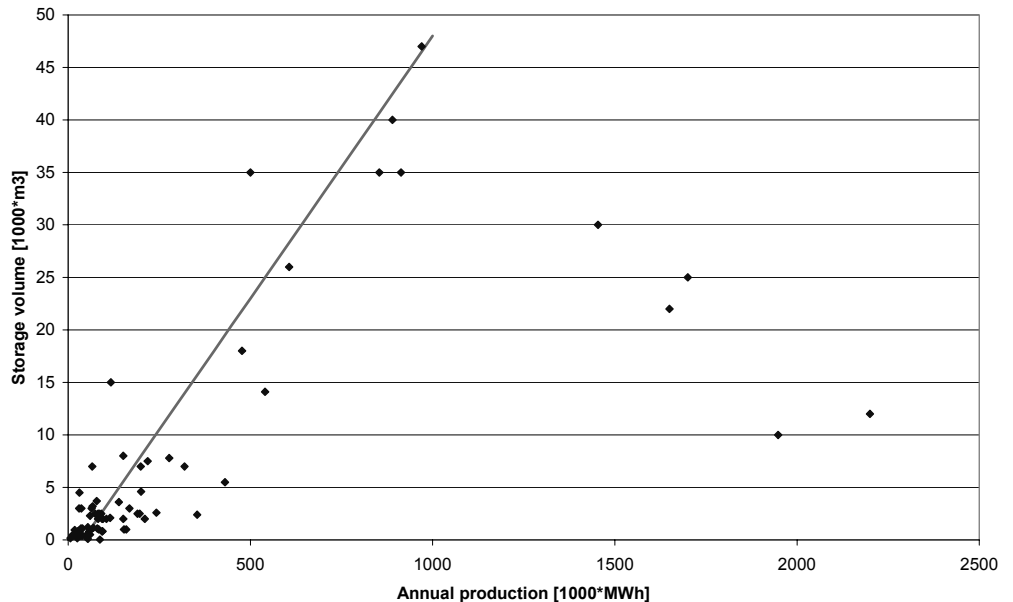Annual production [1000*MWh] (x-axis)

Figure 3-3   Storage dimensioning in the Nordic countries. [Nordvärme 1993]

3.2   Shape of the store

The shape that gives the smallest surface area/volume relations is a spherical store; on the other hand this shape gives a rather large gradient zone when the store is partly charged. Therefore it is an advantage to keep the same cross-sectional area for all heights of the storage.

Two characteristic shapes fulfil this condition:

- Cylindrical shape.
- Rectangular shape.

A rectangular shape is not favourable because surface area/volume is rather large, compared with cylindrical shape. However, a rectangular shape is desirable in cases where space is limited.

A correctly designed store should generate as small a gradient zone as possible; there are four main groups for doing this [Dincer et al 2001]:

- Two-storage model, pressurized or atmospherically.
- Flexible diaphragm, pressurized.
- Labyrinth model, pressurized or atmospherically.
- Stratified model, pressurized or atmospherically.

Pressurized stores may be placed anywhere in the network, atmospheric stores must be placed at the highest point in the network. A pressurized store has no free connection to the surrounding air, only a safety valve to prevent overpressure in the storage. Therefore there is very little risk of oxygen dissolving into the circulating water. On the other hand, a pressurized store may be exposed to negative pressure, e.g. in case of a leak. In this circumstance, there is a risk that collapse may occur.

An atmospheric store has a free connection to the surrounding air and therefore a risk of oxygen dissolving into the water. A continuous supply of oxygen will harm the whole network due to corrosion; therefore oxygen must be prevented from dissolving into the water. This is usually done by installing a vapour pocket or a pocket of any inert gas e.g. nitrogen at the top.

3.2.1   Two-storage model. (Double Vessel System)

This is one obvious way to achieve separation between hot and cold water. In this model two identical stores are built, and when one is full the other one is empty. Therefore there will be

almost no gradient zone. On the other hand this model will require twice as much space as a single store and the investment cost will also be doubled. For these reasons this model is not commonly in use.

3.2.2 Flexible diaphragm.

In this model hot and cold water are separated within the same storage using a physical obstacle. This obstacle can be a flexible membrane or a sliding plate, which has such characteristics that it will automatically place itself in between the hot and cold water.

3.2.3 Labyrinth model.

In this model water is forced to flow through a maze, as if several smaller storages are installed in a serial connection, see Figure 3-4. With external inlet/outlet in number 1 and 7.
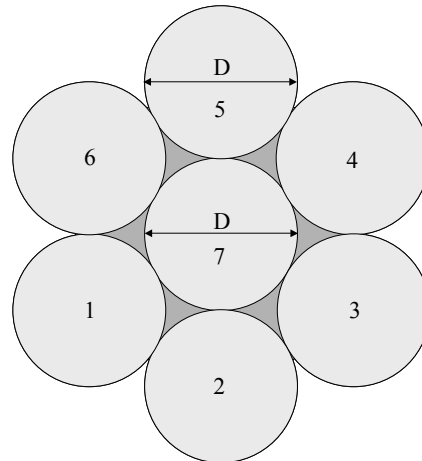

Figure 3-4   Serial connections, in a hexagonal shape.

This constellation gives approximately 13% more surface area adjacent to the surroundings, assuming the same height and the one-storage model have a H/D-ratio equal to 1, H/ D-ratio for this case will be 7/1. Therefore heat loss will also be 13% higher, assuming the same insulation in both cases. The advantages of such a constellation may be a smaller cross-section area, and so a smaller volume in the gradient zone. This will be further discussed in section 3.3.2.

3.2.4 Stratified model.

This type is a single vessel system, which is preferably cylindrical in shape. The advantages with this model are the simplicity, and therefore low investment cost. This is the most common model in use worldwide.

3.3 Deciding height and diameter of a cylindrical storage.

When the volume is determined, height and diameter have to be decided, and therefore several factors have to be investigated:
- Minimum loss of energy.
- Efficiency.
- Available space.
- Size has to comply with local regulations.

3.3.1 Minimum loss of energy.

Minimization of energy loss is simply to minimize total surface area that gives minimum total heat transfer to the surroundings. Surface area of a cylindrical store is given as a function A(r,H), see equation Eq. 3.1.

$$A_{surf}(r,H) = 2\pi r^2 + 2\pi H r$$

Eq. 3.1

With a fixed volume, height can be expressed as:

$$V(r, H) = r^2 \pi H \Rightarrow H = \frac{V}{\pi r^2}$$

If equation Eq. 3.2 is inserted into equation Eq. 3.1, this gives equation Eq. 3.3:

$$A_{surf}(r) = 2\pi r^2 + 2Vr^{-1}$$

Eq. 3.3

Derivation of Eq. 3.3 gives a function with one variable to determine the relations between height and diameter.

$$\frac{\partial A_{surf}}{\partial r} = 4\pi r - 2Vr^{-2}$$

Eq. 3.4

Solutions of equation Eq. 3.4 gives:

$$r = \left(\frac{V}{2\pi}\right)^{\frac{1}{3}}, \; D = 2\left(\frac{V}{2\pi}\right)^{\frac{1}{3}} \Rightarrow H = \frac{V}{\pi}\left(\frac{V}{2\pi}\right)^{\frac{-2}{3}}$$

Eq. 3.5

$$\frac{H}{D} = 1$$

Eq. 3.6

As seen in Eq. 3.6 the best H/D-ratio from an energy loss perspective is equal to 1. Other factors may have different optimal H/D ratios.

In Figure 3-5 and Table 3-1 the effect of variation of H/D-ratio is illustrated. It is interesting to observe that the surface area does not change a lot between H/D-ratios of 0,5 - 2,5. With a H/D-ratio equal to 2,5, surface area has increased by 8,6% relative to minimum surface area. Assuming similar insulation properties in all cases, the changes in area are directly proportional to the changes in loss of energy.
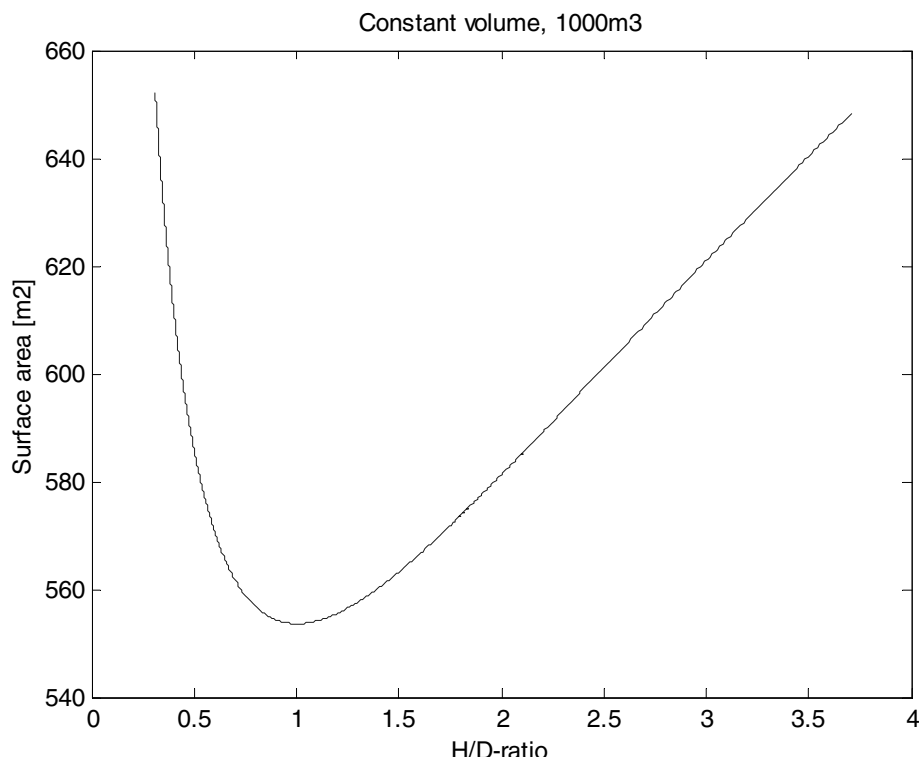

Figure 3-5   Effect of different H/D-ratios, area vs. H/D-ratio.

In both Figure 3-5 and Table 3-1 some assumptions are made:
- Constant volume of 1000 m³.

- Annual average temperature of the store is 75°C.
- Annual average ambient temperature is 5°C.

Insulation used is mineral wool, with conductivity at 0,036 W/(m*K).

Table 3-1   Variations of energy loss, with variation of H/D-ratio and insulation.

H/D-ratio	0,100m	0,150m	0,200m	0,250m	0,300m	Changes [%]
0,5	127,5	85,4	64,2	51,4	42,9	5,8
1,0	120,5	80,7	60,7	48,6	40,5	0,0
1,5	122,6	82,1	61,7	49,5	41,2	1,8
2,0	126,5	84,7	63,7	51,0	42,6	5,0
2,5	130,8	87,6	65,9	52,8	44,0	8,6
3,0	135,1	90,5	68,1	54,5	45,5	12,2
3,5	139,4	93,4	70,2	56,2	46,9	15,7

Heat loss [MWh/year] with different insulation thickness

As a comparison: A normal household in Norway has an energy consumption for space heating and hot water within the range of 15 - 25 MWh/year.

3.3.2   Efficiency.

Efficiency of the store may be defined as:

$$\eta_{acc} = \frac{V_{use}}{V_{tot}}$$

Eq. 3.7

Useful volume of the store is defined as the volume of water with sufficient temperature when fully charged, and the volume of the diffusor is not considered.

As the hot and cold water of the store mix, the useful work potential in the system, or exergy reduces. Loss of exergy occurs because of internal heat transfer from hot water to cold water, this causes dissipation of exergy in the storage, but no loss of energy. Between hot and cold water a layer will develop, which grows with time, with lower temperature than required for the DH network. The simulation of a store with diameter of 10m is shown in Figure 3-6. After 120 hours the layer was approximately 1,16 meters, if defined as an area with less than 97°C and more than 53°C.
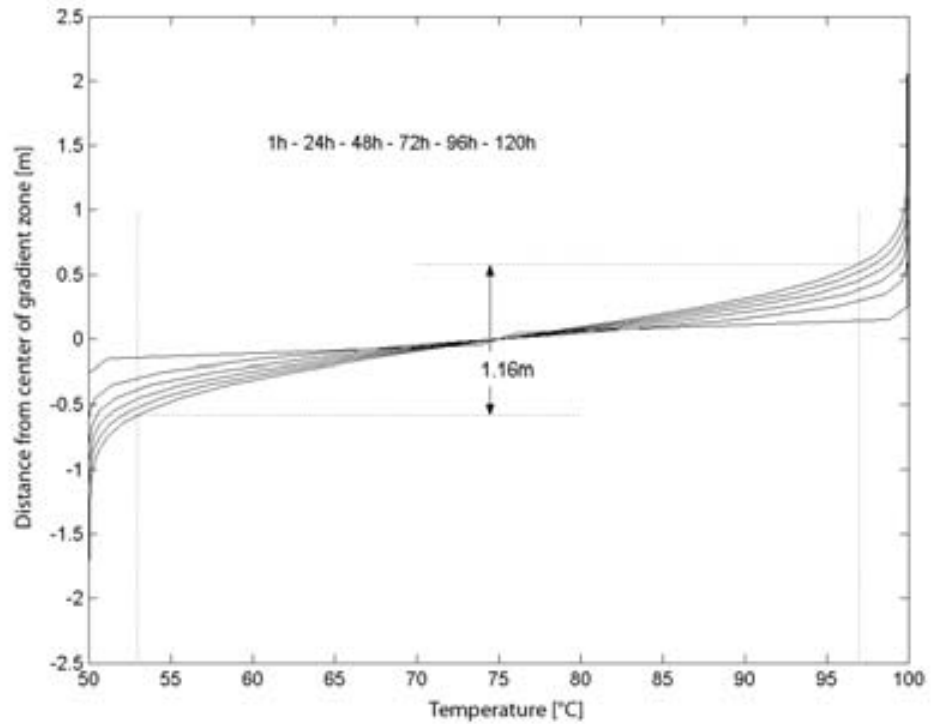
Figure 3-6   Water temperature as function of time, across layer, 1 meter from wall.

Mathematically this zone can be expressed as the volume where:

$$\frac{d\mathrm{T}}{d\mathrm{h}} > \varepsilon\left(\frac{d\mathrm{T}}{d\mathrm{h}}\right)_{max}$$ Eq. 3.8

Eq. 3.8 [Hermansson 1993] expresses an area of the gradient layer where the temperature is out of the acceptable area, defined by ε. In Figure 3-6 it is defined as a volume with a temperature between 53 C° and 97 C°, this gives an ε of (97-53)/(100-50) = 0,88. With an ε = 0,88, the thickness of the gradient zone is 1,16 m. The thickness of the layer is not to any great extent influenced by the diameter, at some distance from the wall. Mainly the temperature difference between hot and cold water is the cause of growth of the gradient zone. Only close to the wall there will occur an extra growth of non-productive volume due to the wall effect, if this area is neglected a smaller diameter is preferable to achieve a smaller non-productive volume and therefore a higher efficiency of the storage. This may explain why there are a concentration of storages with a H/D-ratio among 1,5 - 2,0.

Factors that influence development of the gradient zone:

- Mixing at the inlet.

- Natural convection.

- Heat diffusion and conduction.

3.3.2.1   Mixing at the inlet.

Consider a situation with a full store of cold water and charging is started. This action will immediately cause a mixing of cold and hot water and a gradient zone is initiated. The same situation will occur if the store is filled with hot water and discharging is started. To prevent a large gradient zone two aspects have to be taken into account:

- Position of the diffuser.
  The diffuser should be placed as far up as possible (as far down as possible when a discharge is started from a storage full of hot water) so the hot water has a minimum length to flow to the top. At the same time the diffuser must be fully submerged at all times when the system is in discharge-mode.

- Flow velocity at the diffuser.
  The velocity should be kept at a level such that penetration into cold water is minimized.

To minimize mixing it is of most importance to keep a low inlet velocity. It is shown in several publications that the Ri-number (Richardson's number) is an important measure to determine if the mixing is dependent on the incoming velocity [Goodfellow et. al. 2001].

$$Ri \; = \; \frac{Buoyancy}{Momentum\,gradient} \; = \; \frac{-g\frac{\partial\theta}{\partial z}}{\left(\frac{\partial v}{\partial z}\right)^2} \qquad\qquad Eq. \; 3.9$$

The Ri-number may be expressed as in a modified version, equation Eq. 3.10 [Dahl 1993]:

$$Ri \; = \; \frac{g\beta\Delta TL}{v^2} \qquad\qquad Eq. \; 3.10$$

Where *L* is a characteristic length of the problem. The characteristic length for a cylinder is the diameter.

For Ri-numbers higher than 0.25-0.5 it is expected that the amount of mixing is not very dependent on the velocity of the incoming water. There have been some experimental studies on mixing in a small storage by Hermansson [Hermansson 1993] that shows the result used on Eq. 3.10.

When inserting a diameter of 10 m as the characteristic length, L, for the storage of 1000m^3, and a temperature difference of 50°C, maximum velocity of incoming water is 0.86 m/s and 1.2 m/ s, respectively for Ri = 0.5 and 0.25. This indicates that with a velocity below 0.86 m/s of incoming water, the degree of mixing is not dependent on the velocity.

Some mixing is inevitable when charging or discharging is started. To minimize this it is important to:

- Keep the velocity of the incoming water as low as possible
- Keep the temperature difference between the hot and the cold side of the storage as large as possible.

3.3.2.2   Natural convection.

Natural convection of heat occurs from the surface of the store to air. This effect is relatively small. Considering the case with a storage of 1000 m^3, H/D = 1 and 300mm of insulation from Table 3-1, natural convection is responsible for an average temperature drop at 0.7°C per week. Therefore this effect will not have a large impetus for growing the gradient zone.

The heat transfer from the top of the store to surroundings is often referred to as Bernard convection. When this occurs hot water at the top is chilled, and will cause mixing when sinking down to colder layers in the store. If a hot vapour pocket is installed the heat is delivered from the hot vapour instead of the water, and no chilling of water is present [Hemansson 1993]. Atmospheric stores are often equipped with a hot vapour pocket on the top, both to avoid oxygen dissolving into the water and to prevent Bernard convection.

3.3.2.3   Heat diffusion and conduction

Conduction through the wall generates a velocity field near the wall and therefore causes diffusion in the same area. Conduction within the water volume itself will also cause diffusion. Since the conductivity of steel is approximately 100 times greater than for water it will be of interest to minimize the surface area of the storage.

Simulation of the development of temperature in a tank of 10m is shown in Figure 3-7. The surface area is insulated with a layer of 0.3m rock wool with a constant hot water temperature at 100°C and a constant cold water temperature of 50°C. All lines are isotherms, upper lines indicate water of 97°C and lower lines indicates 53°C. Outdoor temperature is constant 5°C, and the storage is half full of hot and cold water, with a fully mixed layer between with thickness of 0.2m and a temperature at 75°C. During the simulation no charging or discharging is permitted, nor is

the water itself allowed to flow. Heat transfer to the surroundings, conduction in the wall, and convection into the wall from the water and heat transfer within the water is accounted for. The simulation is done with time steps at 1 second for the first hour, 60 seconds for the next 47 hours and 180 seconds for the last 72 hours. A total of 120 hours is simulated.
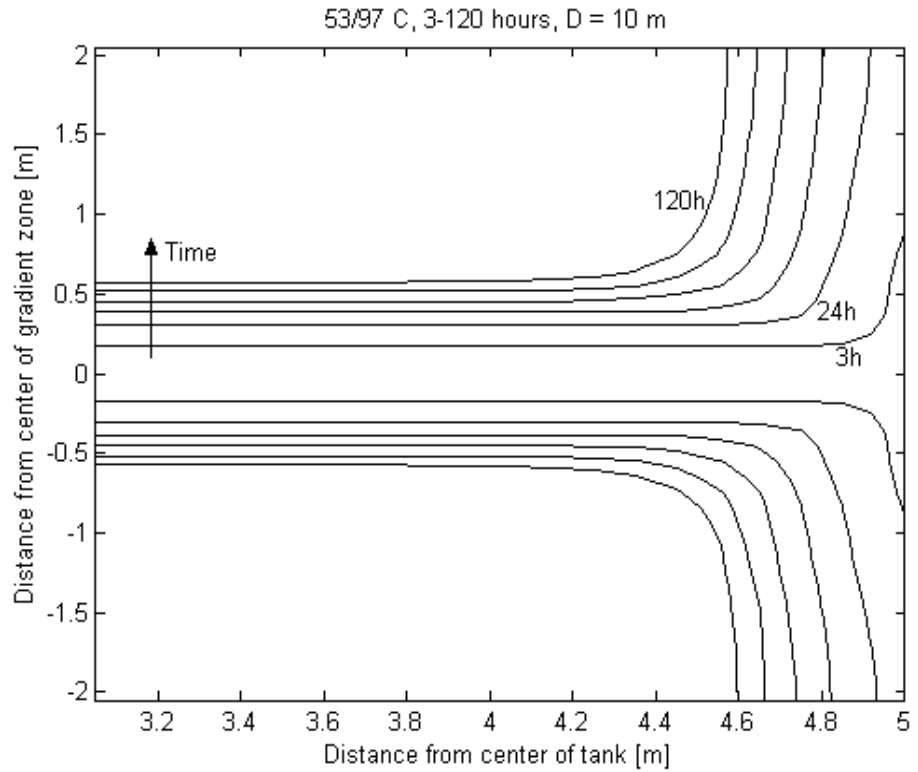

Figure 3-7   Simulated development of temperature in a storage with diameter at 10m, from 3 to 120 hours.

When analysing the results from the simulations two important figures are produced, Figure 3-8 shows how the wall zone develops. For the short time limit it is possible to express the growth of non-productive volume, V_{wall} (volume with temperature lower than 97°C) as a function of the thickness, d_{wall}, that has been linearized, by equation Eq. 3.11.

$$V_{wall} = [D^2 - (D - 2d_{wall})^2]\frac{1}{4}\pi H$$

Eq. 3.11

Where d_{wall} is expressed as in equation Eq. 3.12.

$$d_{wall} = Trendline = 0,0035\,hour + 0,0019$$

Eq. 3.12

From equation 3.12 we can read the rate of growing of the wall zone to be 0,0035m/h.

Likewise for the gradient zone, Figure 3-9 shows how the gradient zone is developing. Also here it is possible to express non-productive volume, V_{grad} (volume with temperature lower then 97°C) as a function of the thickness, d_{grad}, which has been linearized, see equation Eq. 3.13.

$$V_{grad} = (D - 2d_{wall})^2\frac{1}{4}\pi d_{grad}$$

Eq. 3.13

Where d_{grad} is expressed as in equation Eq. 3.14.

$$d_{grad} = Trendline = 0,0066\,hour + 0,4048$$

Eq. 3.14

From equation Eq. 3.14 we can read the rate of growing in the gradient zone is 0,0066m/h. This is approximately twice as fast as the growth of the wall zone.
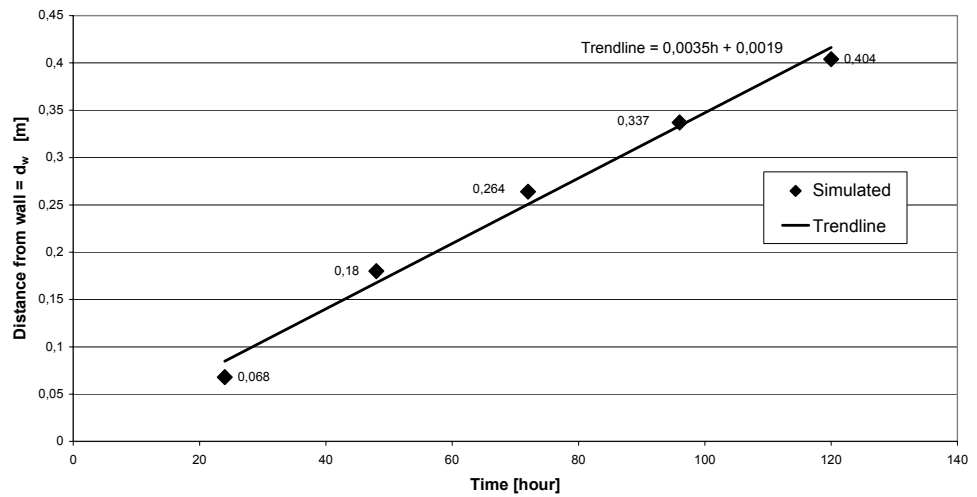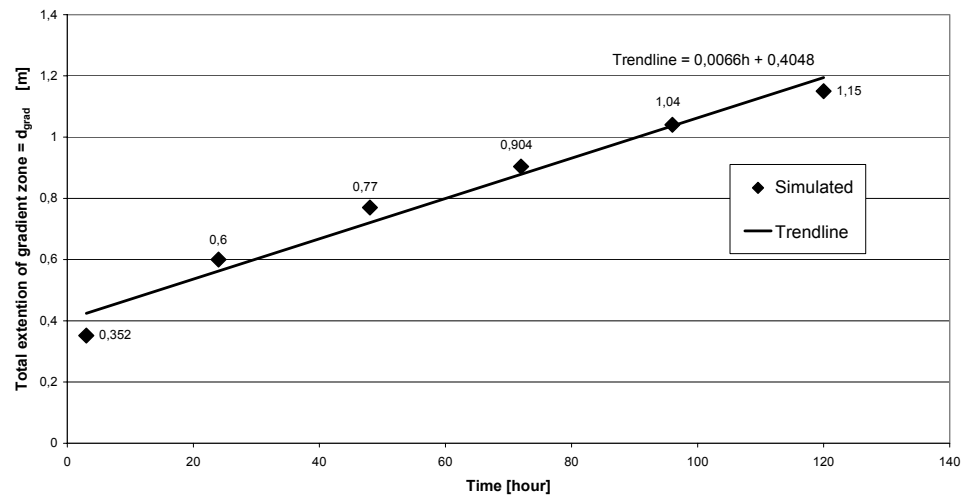
Figure 3-8   Growth of wall zone.

Figure 3-9   Growth of gradient zone.

Due to different growth of wall and gradient zone there will be different optimum of the H/D relation depending on expected storage time. In Figure 3-10 each curve represents extra useless volume relative to minimum non-productive volume with an optimum relation of H/D. For short storage time, 24 hour or less, there will be an advantage of a slender storage tank. The reason for this is mainly due to the gradient zone, caused by mixing from start. With longer storage time it is an advantage with an H/D relation between 1 and 3. In this area there are only small changes in total non-productive volume. These relations in extra non-productive volume are unchanged with different storage tank volume, as illustrated in Table 3-2 and Table 3-3.
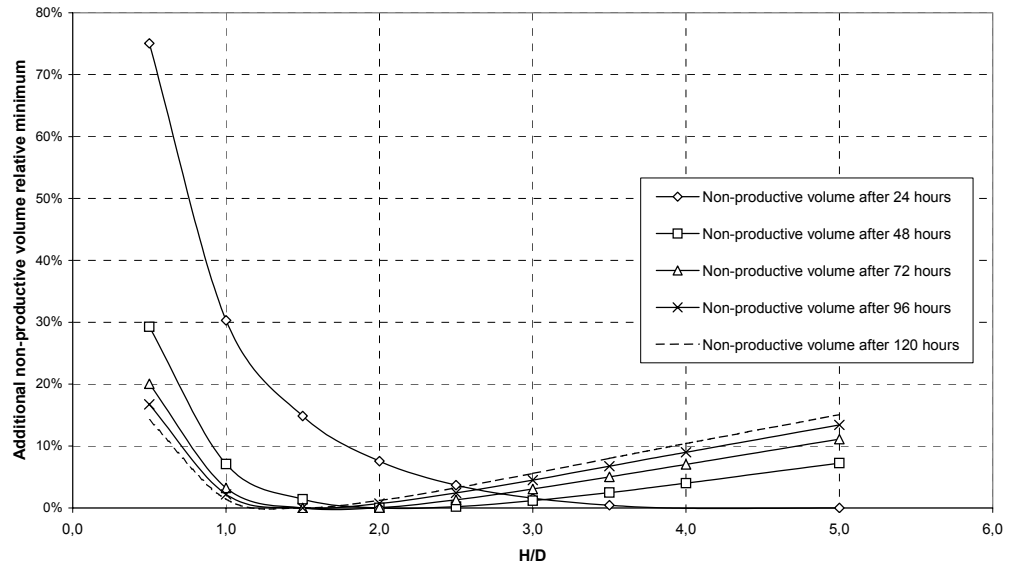
Figure 3-10   *Non-productive volume as a function of time.*

Both Table 3-2 and Table 3-3 shows non-productive volume respectively for total storage volume of 1000 m^3 and 500 m^3, with contribution of both wall and gradient zone.

Table 3-2   Non-productive volumes after 48 hours, total volume of storage is 1000^3.

H/D	V$_{grad}$ [m^3]	V$_{wall}$ [m^3]	V$_{total}$ [m^3]	Relativ change
0,5	106,9	52,0	158,9	29 %
1,0	66,4	65,3	131,7	7 %
1,5	50,2	74,6	124,8	1 %
2,0	41,1	81,9	123,0	0 %
2,5	35,2	88,1	123,3	0 %
3,0	31,0	93,5	124,5	1 %
3,5	27,8	98,3	126,1	2 %
4,0	25,3	102,7	128,0	4 %
5,0	21,6	110,4	132,0	7 %

Table 3-3   Non-productive volumes after 48 hours, total volume of storage is 500 m^3.

H/D	V$_{grad}$ [m^3]	V$_{wall}$ [m^3]	V$_{total}$ [m^3]	Relativ change
0,5	66,4	32,7	99,1	29 %
1,0	41,1	41,0	82,1	7 %
1,5	31,0	46,8	77,7	1 %
2,0	25,3	51,3	76,6	0 %
2,5	21,6	55,2	76,8	0 %
3,0	19,0	58,5	77,5	1 %
3,5	17,0	61,5	78,5	2 %
4,0	15,5	64,2	79,7	4 %
5,0	13,2	69,0	82,2	7 %

3.3.3   *Available space and local regulations.*

In many cases dimensions of the storage are determined by available space, and/or by local regulations. In cases where storage is to be built in a residential environment, local regulations may not allow stores above a specific height.

In some cases it is therefore preferable to build two, or more, smaller stores, although two stores in most cases will give a larger total surface area than one of equivalent total volume.

A pure energy consideration suggests that H/D should be equal to 1,0. However, if the most important issue is to minimize non-productive volume, the storage time must be considered. For a short average storage time, 24 hour or less, there will be an advantage of a slender storage tank.. To determine which ratio that is best it is necessary to apply economic costs of energy and loss of entropy.

In addition to whatever H/D-ratio is selected when considering minimizing loss of energy and loss of entropy, some height must be added. The diffuser at top and bottom requires some space and this volume is also not available for heat delivery. For stores with a steam pocket at top, further height must be added. This will result in a slightly higher H/D-ratio.

This is quite consistent with existing storages in the Nordic countries, where a majority is within a range of H/D-ratio of 1,0 - 2,0.

3.4   Investment

In [NE 1980] a maximum specific investment cost is derived based on the difference in energy cost between a thermal storage and a peak load unit. The derivation is reproduced below.

Annual cost for a thermal storage:

$$C_s = a c_{s,i} E_{s\,max} + \frac{c_{s,ch}}{\eta_{acc}} E_{totl} + D_1 \qquad \text{Eq. 3.15}$$

where a is the investment installment constant, $c_{s,i}$ is the specific investment cost, $E_{s\,max}$ is the maximum storage energy content, $c_{s,ch}$ is the charge energy cost, D_1 is the maintenance cost, η_{acc} is the storage efficiency and E_{tot} is the annually delivered energy from the storage.

Annual cost for an alternative peak unit:

$$C_b = a c_{b,i} Q_{b\,max} + \frac{c_{f\,b}}{\eta_b} E_{tot} + D_p \qquad \text{Eq. 3.16}$$

where a is the investment instalment constant, $c_{b,i}$ is the peak unit specific investment cost, $Q_{b\,max}$ is the maximum capacity, c_{fb} is the peak unit fuel cost, D_p is the maintenance cost, η_b is the efficiency and E_{tot} is the annually delivered energy from the peak unit.

The maximum specific investment cost for the store may now be derived by setting the annual cost for the store equal to the annual cost for the peak load. If the maintenance cost is ignored, the store utilization time is defined as $E_{tot}/Q_{b\,max} = \tau$, and the number of annual storage cycles is defined as $E_{tot}/E_{s\,max} = N_s$, the maximum specific investment cost for the storage may be expressed as:

$$c_{s,i\,max} = \left[\frac{c_{b,i}}{\tau} + \frac{1}{a}\left(\frac{c_{f\,b}}{\eta_b} - \frac{c_{s,ch}}{\eta_{acc}} \right) \right] N_s \qquad \text{Eq. 3.17}$$

In Figure 3-11 an example of the maximum specific investment cost is plotted as a function of the energy cost difference $\dfrac{c_{f\,b}}{\eta_b} - \dfrac{c_{s,ch}}{\eta_{acc}}$.
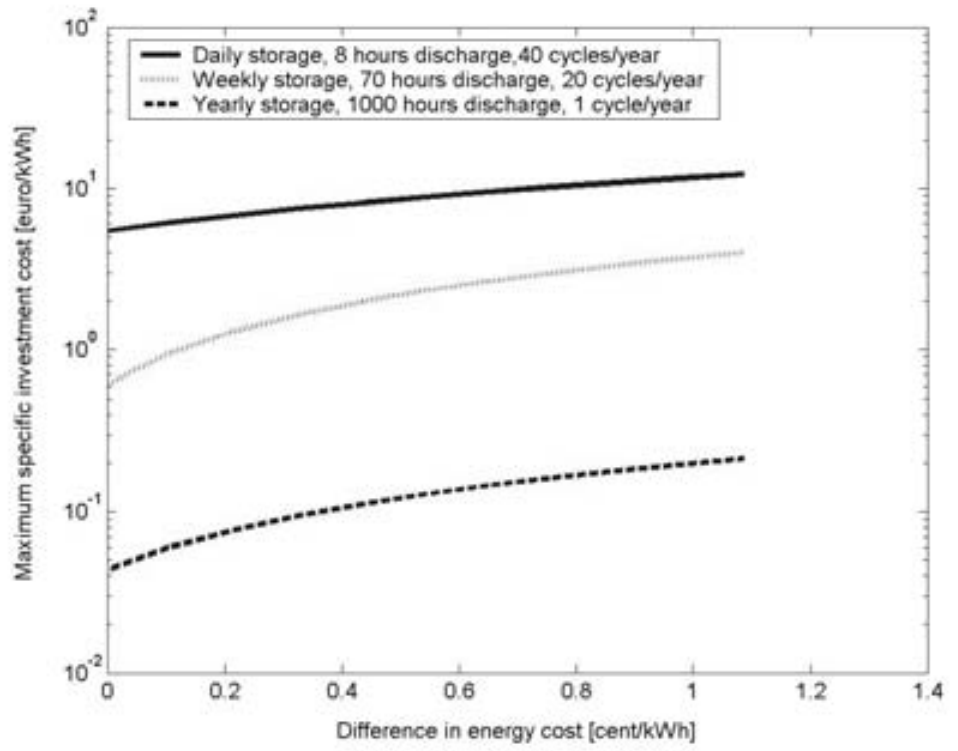
Figure 3-11.   Maximum motivated specific investment cost for three types of thermal storages. [NE 1980]

4 Thermal storage optimisation

4.1 DH&C operational optimisation survey

The problem of optimising the planning and operational scheduling of district heating plants has been addressed since the early 1980s. Although the overall objective has been to find the most economic way to satisfy a district heat demand, various models, optimisation methods and points of view have been suggested.

Due to the scope of the optimisation different time horizons are considered. For the planning of plant capacity investment as for the planning of district augmentation an appropriate time horizon would cover several years whereas the perspective for optimum operation of a plant normally considers days or even hours. The latter category, the short term planning problem, is the one studied in this survey.

The classic problem in the theory of optimisation is formulated below:

$$\min_{X} \quad c = f(X) = f(x_1,...,x_n)$$

$$subject\ to \quad g_i(X) = 0 \quad i = 1,...,m$$
$$h_i(X) \leq 0 \quad i = 1,...,k$$

Eq. 4.1

where the components of the vector X are called decision variables, the function f the objective function and the the set of conditions $g_i(X)=0$ and $h_i(X)\leq 0$ problem constraints.

Depending on how the problem is formulated; linear/nonlinear, continuous/discrete, deterministic/stochastic, there exists a number of methods for solving the problem:

- heuristic methods
- linear programming (LP)
- mixed integer programming (MIP)
- dynamic programming (DP)
- nonlinear programming
- stochastic programming

The short term planning problem may be divided into three components; unit commitment, load dispatch and supply temperature control. In the *unit commitment problem* the objective is to find the most economically suitable combination of energy conversion units according to e.g. heat demand, start-up and shutdown costs. In the *load dispatch problem* the objective is to find optimal production plans for all engaged units as well as optimal purchase plans for primary and electric energy. In the *supply temperature control problem* the issue is to control the supply temperature such that the overall production cost is minimised.

4.1.1 Operational models

As has been already mentioned a variety of models and methods for operational optimisation have been developed. In this section we will describe a few. For a more extended literature survey see e.g. [Dotzauer 2001].

4.1.1.1 Heat and Power Production Units

In a *heat boiler*, heat is produced by combustion of fuel or by electrical heaters. Solid fuel is normally used for base load production whereas oil or gas is used for peak load. Boilers may be categorised in accordance with the heat carrier connected; hot water boilers and steam boilers are two examples of heat boilers. Steam boilers are normally more expensive than hot water boilers but connected to a steam turbine a steam boiler may be extended for power production. [Fredriksen, Werner 1993]

In *cogeneration plants* i.e. plants producing both heat and power, the fuel utilization is much higher compared to pure electric power generation due to the fundamental fact that all thermal energy may not be converted into work. The ratio between electric power produced and produced heat is called the alpha value and is defined as: $\alpha = P/Q$. In *back pressure plants* the alpha value is kept at its minimum producing as much heat as possible. The relation between generated heat and power is almost linear and is called the back pressure line. The name back pressure stems from the fact that the steam is condensed at a higher temperature than in a condensing power plant giving an increased condenser pressure.

For a back pressure plant the electric power generated is strictly dependant on the amount of heat requested. On a market with varying electricity price this is of course a major drawback. However, completing the back pressure unit with a low pressure turbine and leading part of the steam through a cold condenser the alpha value may be arbitrarily varied. This type of plant is called an *extraction plant*. In Figure 4-1 the possibilities for cogeneration are outlined.
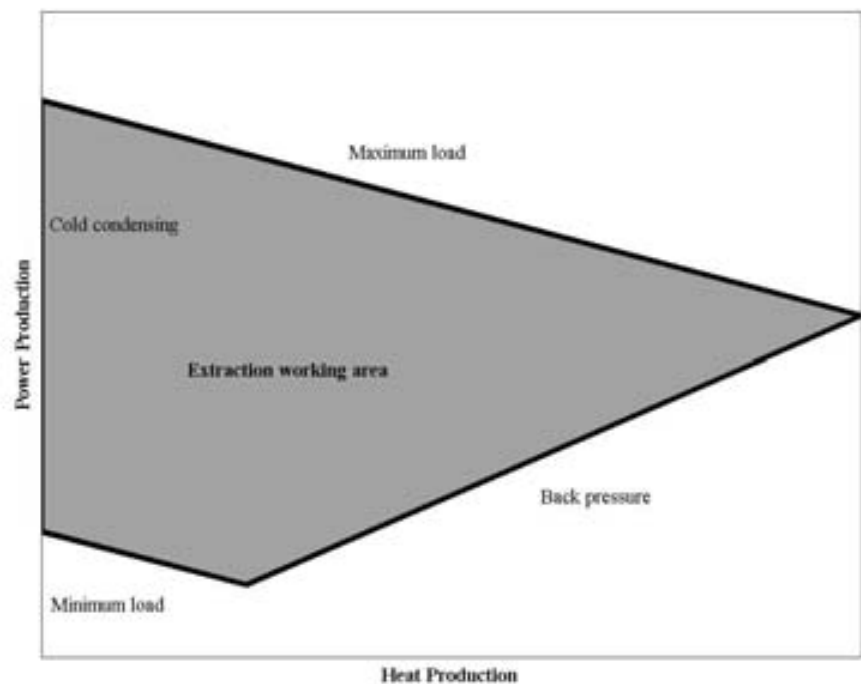

Figure 4-1. Possible combinations for steam turbine CHP cogeneration.

In [Ravn, Rygaard 1993] the non-linear model derived comprises back pressure units, extraction units, peak units and a pressureless water tank for storage. In this model the total running cost includes startup and production costs for all the units together with the purchased electric energy. The production cost is a piecewise 3rd order polynomial of one independent variable with discontinuities due to the fact that the fuel composition varies as the plant energy output increases.

For peak units the production cost depends only on the heat produced. This is also true for the back pressure model as the electric power is given by the back pressure line. The back pressure line is given by a 2nd order polynomial. For the extraction plant the production cost depends only on the electric power produced as the heat produced is related to the electric power via the iso-fuel lines. The iso-fuel lines are given by a 2nd order polynomial.

Production levels, storage tank content and time are all discretised and dynamic programming is used for solving the operational optimisation problem (unit commitment, load dispatch). The time horizon for the problem is 48 hours.

In [Nilsen 1994] lagrange multiplier equations are used for solving the load dispatch problem at each timestep. The unit commitment problem is then solved using dynamic programming over the complete period. Back pressure units and extraction units are represented by linear back pressure lines and linear iso-fuel lines respectively. Production costs for all units are second order poynomials.

A mixed integer linear programming model is used in [Eriksson 1994]. The model covers unit commitment and operation of heat storage devices over a time period of one to several days. Optimal operation is found using a branch and bound method.

Two methods to represent start and stop costs of a production unit have been found in the literature. The first method used in e.g. [Nilsen 1994] is using a fixed cost for starting and stopping a production. In the second method used e.g. in [Dotzauer 1997], the start cost depends on the time period the production unit has been out of use, $T_{shutdown}$.

$$c_{start} = c_{start1} + c_{start2}\left(1 - e^{-C_{start3}T_{shutdown}}\right)$$ Eq. 4.2

4.1.1.2 Thermal stores

The most commonly used storage model for operational optimisation is the one derived in [Ravn, Rygaard 1993]:

$$E_s(t+1) = (1-\alpha)E_s(t) + Q_s(t) - \beta(t)$$ Eq. 4.3

where E_s denotes the storage energy content, α the heat loss to the surroundings proportional to energy content, Q_s the energy charged into the tank and β the constant storage tank loss.

A storage tank model comprising net dynamics was derived in [Zhao et al 1998]. However, one of the conclusions was that the net dynamics did not play a vital role for the operation of the tank. The supply temperature should be kept as low as possible in order to reduce the fuel consumption.

In [Zhao 1995] and [Wigbels et al 2002] the DH-network is used for storage.

4.1.1.3 State at the end of an analysis period

If the storage energy content at the end of the period investigated is not treated separately, all optimisation methods will end up with the storage being empty. In [Eriksson et al 1989] three methods for avoiding this are discussed:

- an estimated value of the heat content for the final period is added to the cost function
- a specific energy content is required in the final period.
- extending the planning horizon and therby reduce the influence of the final period

For the method of adding an estimated value of the heat in the final period to the cost function, this value should be based on the marginal cost of heat production in the time period following the period investigated.

Also in [Dotzauer 1997] the final content of the storage is included in the cost function.

4.2 Type of storage problem

A physical storage is first of all a means for improving the operational costs of the district heat or cold production. The need for a store rises from two major reasons:

1. Utilisation of the capacity limits of the production units (especially between base and peak load units)
2. Co production of heat and power
3. Heat or cold power shortage in certain parts of the system, often due to certain bottlenecks and/or extraordinary user patterns

The two first types of storage problem are an economical optimisation regarding total costs of operation and capital costs for the plant. An economic optimum storage size or capacity is to be defined. For the third problem storage can be a solution to solve a heat or cold delivery problem, i.e. a local capacity problem. The latter is thus a question of storage loading and unloading capacity and the storage solution costs must be compared with the total cost introduced by an extra, local heat or cold production unit. For this problem the decision methodology presented in chapter 5 is suitable for calculating the utilisation of the storage.

4.3 Type of optimisation problem

The optimisation of a certain plant layout implies an optimisation with respect to the total cost of running the plant. This is often mixed up with *operational optimisation*, that is to "produce" the heat , electricity or cold as efficiently as possible. To find an optimum solution both firm costs and operational costs are part of the problem. For a certain solution which is tested, the operational costs must be calculated. To calculate these costs representatively, optimum operation versus the operational constraints and input data is assumed. Thus, for every investigated solution an optimum operational cost and capital/firm costs needs to be found since the constraints of the operation change as the layout of the solution changes. This is further discussed on a general basis in [Nielsen 1996].

In this report the other parts of the DH&C system is considered fixed. Only the size or capacity of the store is to be found. A common way to establish a solution to these optimisation problems is efficient search methods that are handling the changes in storage capacity as an outer optimisation loop.

The optimization curve for these kinds of analysis problems is typically very flat (if there is an optimum at all!). Therefore it is important to visualise the sensitivity of the optimum size. An exhaustive search reporting the costs for incremental intervals of storage size is thus preferable compared to more efficient search methods that inevitably will hide more information [Nielsen 1996].
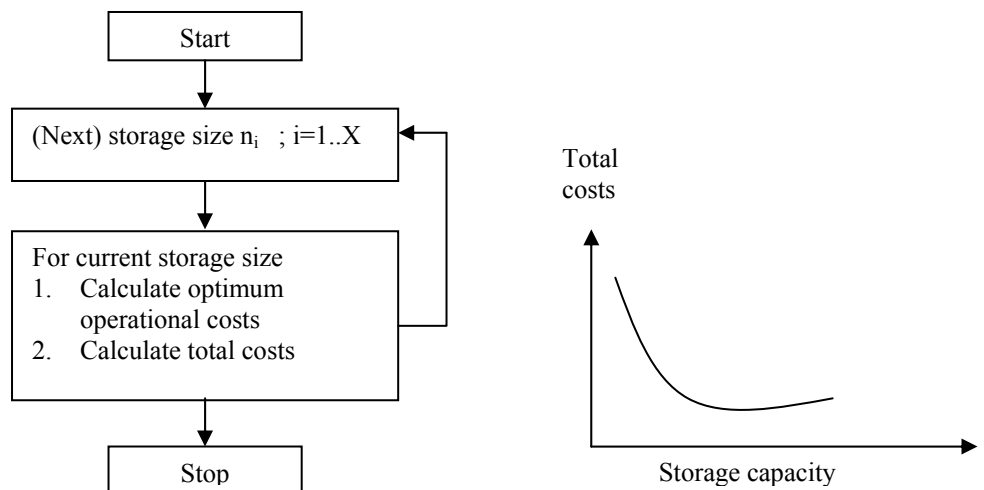

Figure 4-2. The optimisation problem.

4.4 Constraint scenarios and optimisation uncertainties

When defining and solving optimisation problems, assumptions and simplifications are most often inevitable. These undesirable elements imply uncertainties in the final result that must be considered before subsequent conclusions are drawn. Also future uncertainties due to changes in energy prices and district heat load have to be considered when interpreting the optimisation result. The sequel of this section will discuss how to analyse some of the uncertainties related to the optimisation methods presented in this report.

4.4.1 Uncertainties in input data

Heat load and electricity prices are crucial parameters for the optimisation methods presented in this report. One of the incentives for installing storage in combination with CHP back pressure plants is to enable the electricity production to follow the electricity price. The greater the electricity price fluctuations the greater the gain from a storage.

There are at least two important reasons for analysing uncertainties in input data. To avoid the optimisation result to be dependent on the current conditions for a specific year, it is important that

the input data is representative for the actual dimensioning case. If possible, the optimisation should be performed on time series from several different years.

The second and perhaps most important reason for uncertainty studies is the future development in energy prices as well as district heat load and plant capacity. The most straightforward way to study future development is to run the optimisation based on model parameters and modelled time series representing possible future scenarios.

One way to generate future district heat load time series is to simply scale the present heat load profile. This is a crude estimation of the future load profile development but nevertheless it may give useful knowledge about the how the decision and optimisation results are affected by an increasing heat load. In chapter 6.8.1 this method is used for studying how the relation between the base capacity and the the maximum heat load is affecting the optimal storage volume.

Another way to generate heat load profiles is to model the load as a piece wise linear function of the outdoor temperature adding the daily and weekly fluctuations due to tap water consumption.

4.4.2   Uncertainties in assumptions

One of the assumptions made in the optimisation methodology presented in chapter 6 is that the plant is optimally operated, i.e. the operation of the plant is supervised by a production planning tool. If the plant is suboptimally operated it will no longer fully utilize the dimensioned storage volume. Running the dimensioning methodology presented in chapter 6 on suboptimal operational rules, will result in a smaller storage volume since a suboptimal operation is utilizing the storage facilities to a lesser extent.

Another assumption is that all excess electric energy may be delivered to the electricity grid. If this is not the case the storage volume will most likely be overestimated. Unfortunately there are no appropriate means for analysing this situation other than introducing an extra optimisation constraint.

4.4.3   Methodology uncertainties

The dimensioning methodology presented in section 6 uses models to describe the behaviour of heat storage, boilers, CHP units etc. Since every model describing a real process is a simplification, the output from the model will to some extent deviate from the real output. The model discrepancies will therefore also affect the result of the dimensioning methodology i.e. the optimal storage volume. Analysing this sort of uncertainty requires knowledge about the model discrepancy i.e. what simplifications are made? Non-modelled properties that would limit the storage utilization implies an overestimated storage volume.

Other uncertainties connected to the methodology are those occurring from discretisation. Both the decision and the dimensioning methodology use discrete time series. When choosing the length of the sample period the dynamics of all relevant components should be considered. For instance the dynamics of district heat load and the dynamics of the electricity market are both in the range of a few hours hence a sample period of 1 hour should be applicable.

In the dimensioning methodology the energy content of the heat storage is discretised. In chapter 6.8.2 it is shown how the discretisation affects the optimal volume.

5 Decision model based on historical data

5.1 Introduction and description of test cases

There are two different test cases that are used in the development of the test model, one in Norway and one in Sweden. The test cases are chosen based on availability of historical data and their interest in a thermal storage for smoothing out the thermal load during a day.
The district heating plant in Norway is a heat plant and has very little electricity production. The district heating plant in Sweden is with both heat and electricity production.

5.2 Objectives and limitations of the methodology

The objective of the decision model based on historical data is to develop a simple and straightforward model with deterministic data as a first investigation around the economical relations. The output from the first step in the model is only an economical quantification of the benefit that is possible to get with different size of the storage.There are several reasons for this approach.

It is very difficult to run a thermal store with an optimal operation under uncertainty. And the physics are complicated to model with sufficient accuracy. There is uncertainty around the heat loss and energy prices and the models for the store efficiency and thereby the heat loss is complicated to evaluate.

Under real operation it is necessary to have a forecast of the heat load, and the energy price for the considered time period before it is possible to decide how to operate the storage. The operational staff, or preferably a computer program, will have to control the operation of the store based on the prediction of future heat load and energy prices. With a partially filled tank and flow in and out of a thermal storage, it is necessary to calculate the thermal stratification and this calls for models at a micro level. The heat loss itself is easy to calculate with a known temperature profile in the storage. However, the temperature profile is not so straightforward. The exergy loss will also depend on the thermal stratification.

5.3 Development of analysis methodology

The main idea behind this decision model, which is based on historical data, is that there are different energy prices for different heat production units and that smoothing out the thermal load during a day will make it possible to continue to operate the plant without starting a more expensive heat source. Three of the cases that are studied have a base heat production based on refuse incineration. During days where the heat load periods are fluctuating around the maximum heat production from the refuse incineration it is obvious that it is possible to reduce the fuel cost by storing heat during periods where the heat load is below the production capacity from the refuse incineration and use the stored energy when the heat load is above the production capacity.

With this deterministic approach it is possible to count the number of days where the heat load exceeds the limits in the production capacity with different production cost. Within these days it is possible to reduce the fuel cost by storing energy produced with the cheapest production unit.

By calculating the amount of energy above and below the actual production limit it is possible to determine the necessary size of the store.

With a given store, it is also possible to use it on days where the average load is above the base load but where the heat load during some hours is below the base load.

In the case of combined heat and power the store is used to shift the electricity production in time to maximize the production to timeslots with high electricity price. The excess heat produced during these periods is stored and used when the electricity price is low.

5.4 Calculation method

5.4.1 Base load only

The first step is to find the days where the average heat load is below the actual production limit:

Average heat load:

$$\overline{Q_L} = \frac{1}{n}\sum_{i=1}^{n} Q_i \qquad \text{Eq. 5.1}$$

Average heat load criteria:

$$\overline{Q_L} < Q_{prod} \qquad \text{Eq. 5.2}$$

The next criteria is that the heat load during the period is above the production limit

$$Q_{prod} < Q_i \qquad \text{Eq. 5.3}$$

For periods where both Eq. 5.1 and Eq. 5.3 are fulfilled, it will be possible to cover the total heat load with use of the base load.

Storage criteria

$$\overline{Q_L} < Q_{prod} \cap Q_{prod} < Q_i \qquad \text{Eq. 5.4}$$

If the storage criteria in Eq. 5.4 is fulfilled the necessary heat storage for this period is given by:

Heat storage:

$$Q_{s,j} = \sum_{i=1}^{n} Q_i \in Q_i > Q_{prod} \qquad \text{Eq. 5.5}$$

If the heat load is above the production limit during several periods separated by periods where the heat load is below the production limit, Eq. 5.5 has to be modified to take account for this.

For a given period and a given production limit it is then possible to find the actual days and the corresponding storage volume that is necessary to avoid starting another boiler.

5.4.2 Increased utilization of base load

For days where Eq. 5.1 is not fulfilled but the heat load during the day is below the base load it is possible to store energy from the base load and reduce the use of more expensive heat production.

$$Q_{prod} > Q_i \qquad \text{Eq. 5.6}$$

The heat storage is calculated in the same manner as Eq. 5.5 with the same conditions.

$$Q_{s,j} = \sum_{i=1}^{n} Q_i \in Q_i < Q_{prod} \qquad \text{Eq. 5.7}$$

For a given period and a given production limit it is then possible to find the actual days and the corresponding storage volume that is necessary to maximize the utilization of the base load.

5.4.3 Combined heat and power

The electricity price will vary during the day for all days during the year and it is theoretically possible to increase the income for all days. Therefore a minimum limit is applied to the difference in electricity price, to check that the variations is above a given limit.

Price variations criteria:

$$\max(c_e) - \min(c_e) > c_{e\lim} \qquad \text{Eq. 5.8}$$

There is also a limitation in the average heat load during the day. If the amount of heat that is possible to shift is very small it is probably not worth considering.

Energy criteria:

$$\overline{Q_L} > Q_{L\lim}$$

Eq. 5.9

For a given period where Eq. 5.8 and Eq. 5.9 is fulfilled the necessary storage volume is calculated by sorting the electricity price for the next 24 hours and produce at maximum power in the hours with highest price until the stored heat will cover the requested heat load.

5.5   Historical data

Figure 5-1 shows the load for the heat production only district heating company from 01.05.2002 to 01.05.2003
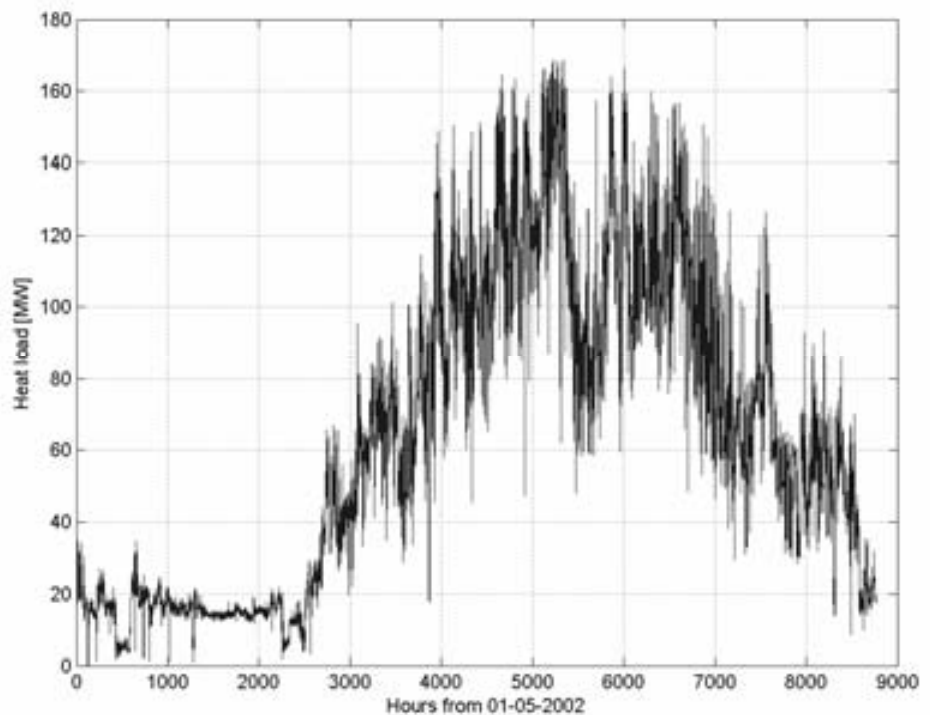

Figure 5-1 Heat load profile heat only case

The production limit for the base load is 65 MW consisting of biomass and refuse incineration.

Figure 5-2 shows the load for the combined heat and power district heating company from 01.01.2002 to 31.12.2003
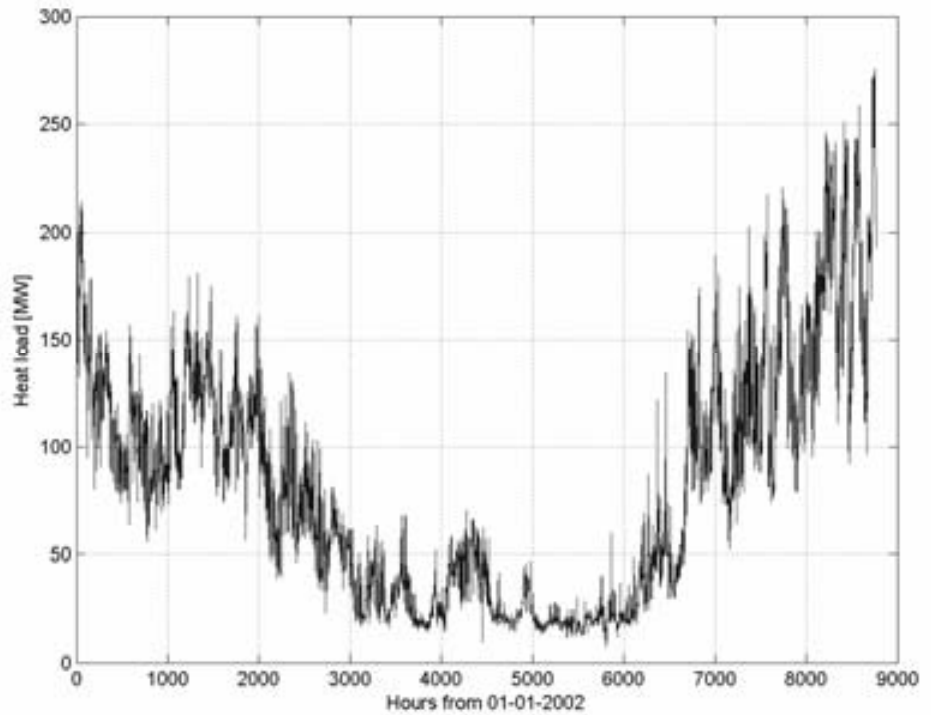
Figure 5-2 Heat load profile heat for combined heat and power case

5.6 Results from demonstration of methodology

5.6.1 Base load only

By using Eq. 5.1 to Eq. 5.5 we find that there are 43 days with average daily load below 65 MW and a daily maximum load above 65 MW. For these days the system could be operated by base load only and it is obvious that the storage could reduce the fuel cost.

Within these days the necessary stored energy varies between 0 and 280 MWh. as shown in Figure 5-3, corresponding to 0 to 6000 m^3. Depending on the size of the storage it is possible to cover up to 1.45GWh with base load instead of oil or electricity.
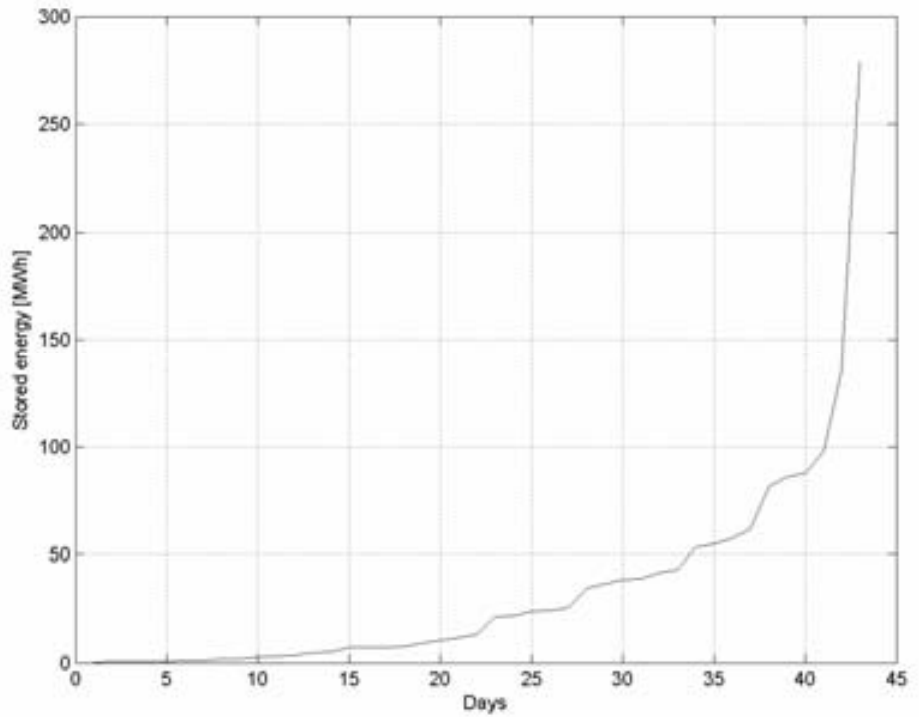
Figure 5-3. Stored energy with base load only

5.6.2 Increased utilization of base load

By using Eq. 5.6 to Eq. 5.7, we find that there are 42 days where the heat load in some hours is more than 5 MW below the limit on 65 MW. During these hours it would be possible to produce to the storage. The extended use of the base load for these hours is 2.4 GWh.

During these days the stored energy varies between 0 and 145 MWh as shown in Figure 5-4 corresponding to 0 to 3000 m^3. Depending on the size of the storage it is possible to cover up to 2.4 GWh with base load instead of oil or electricity.
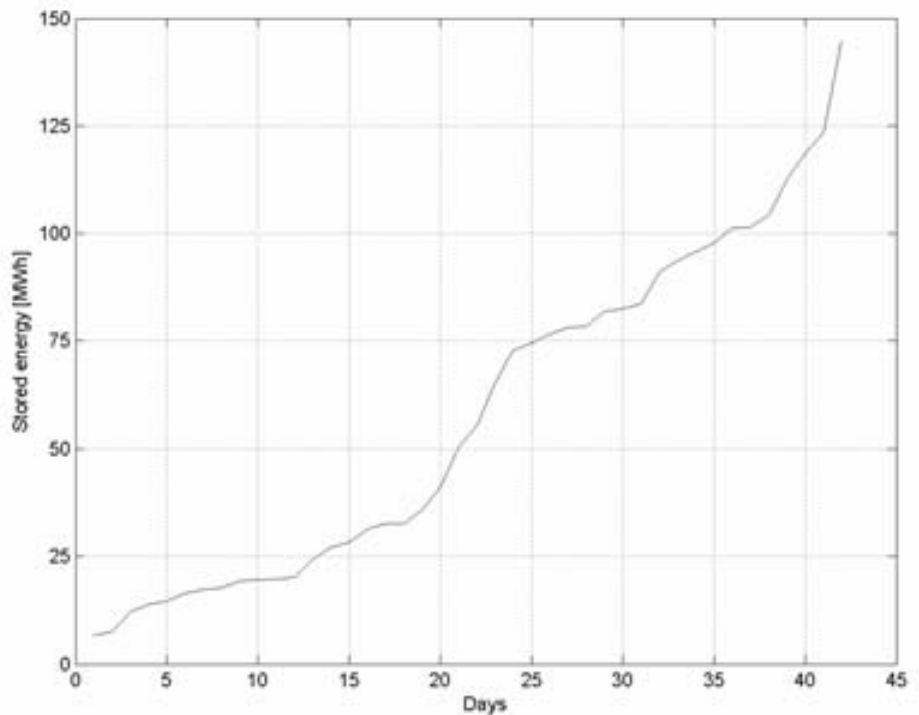
5.6.3   Combined heat and power

By using the method described in Chapter 5.4.3 on the data shown in Figure 5-2 it is possible to find the necessary heat storage to maximize the income from electricity sale.
By setting the limits described in Eq. 5.8 and Eq. 5.9 to zero the required storage to maximize the income from electricity sale is shown in Figure 5-5. The stored energy varies between 70 and 1745 MWh corresponding to 1500 to 37 000 m^3.

The energy content that is run through the store is 2 372 MWh and the income from electricity sale is increased with approximately 280k€.
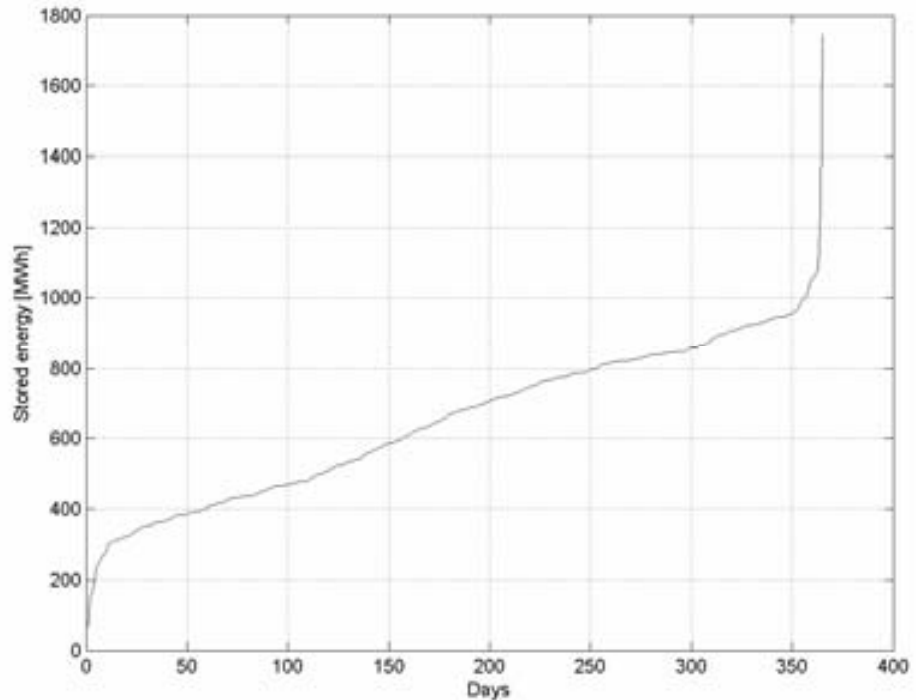

Figure 5-5 Required heat storage for optimizing income from electricity sale

5.7   Conclusion

A method for calculating the economic benefits by using heat storage in different district heating system is presented and demonstrated. The method is based on historical data. The calculations for some cases shows that the method is applicable as a first approach for an investment decision in heat storage.

6 Optimisation model and methodology for optimum thermal storage capacity

6.1 Introduction

The size of the heat store affects the operating cost due to a number of reasons. A large storage tank enables flexible storage management whereas a smaller tank has less heat loss to the surroundings e.g. a large heat store enables a back pressure plant to control the electricity production according to the electricity spot market price, to a larger extent than a smaller heat store. The larger the store, the buffer between heat supply and demand and therefore the greater the operational flexibility for the heating plant.

This chapter presents a methodology for finding the optimum size of a heat storage for a given district heat plant. Operational optimisation models found in relevant publications are used to optimally operate the given plant together with a heat storage such that the total operating cost is minimised. The models used for operational optimisation are presented followed by a description of the complete optimisation procedure and the inputs required and the chapter ends with a demonstration of the optimisation concept.

Dynamic programming is engaged for finding the optimal unit commitment, i.e. how to optimally employ the production units but also how to utilize the heat storage. Lagrange multilplier method is used to solve the load dispatch problem, i.e. to find the optimum heat production distribution among the active production units.

6.2 Objectives and limitations of the model and methodology

The objective is to present a methodology for finding the most economically suitable heat storage size for a given district heat plant. The economic analysis is limited to running and investment costs and issues regarding tax regulations will not be addressed. Further, the models presented in this chapter does not include maintenance cost. The economic analysis is also limited by the fact that the equipment life time is not taken into account i.e. the unit commitment and load dispatch derived by the operational optimisation methodology may not be optimal in the sense of maximum equipment life time.

6.3 Model representation of heat and power production

6.3.1 Conclusions from literature survey

According to [Zhou et al 1998] the net dynamics are not crucial when studying the effects of locating a heat store to a CHP/DH plant. Therefore the net dynamics are neglected and the heat storage is modelled in terms of energy flow rather than dynamic temperature levels and mass flow.

Choosing production unit models and an operational optimisation method is a matter of balancing the need for accuracy with simplicity. The Lagrange multiplier method and dynamic programming applied to a second order cost function is estimated as appropriate for the currrent purpose. The Lagrange multiplier method solves the continuous part of the problem and the dynamic programming the discrete part.

The models used for operational optimisation are described in the preceeding chapter and are to a great extent influenced by [Benonysson 1991], [Nilsen 1994] and [Dotzauer 1997].

6.3.2 Representation of production units

6.3.2.1 Start and Stop Costs

Both start and stop costs are modelled by a fixed cost regardless of the time the unit has been out of use.

6.3.2.2   Boiler

The running cost of a boiler is determined by the electricity usage and the fuel consumption relative to the amount of heat produced.

Fuel consumption:

$$f_b = f_{b0} + f_{b1}Q_b + f_{b2}Q_b^2 \qquad \text{Eq. 6.1}$$

Electricity usage:

$$P_{help,b} = e_{b0} + e_{b1}\frac{Q_b}{Q_{b\max}} + e_{b2}\left(\frac{Q_b}{Q_{b\max}}\right)^2 \qquad \text{Eq. 6.2}$$

Running cost = fuel consumption + electricity usage:

$$c_b = T\left(f_b c_{f_b} + c_e P_{help,b}\right) = T\left(c_{f_b}\left(f_{b0} + f_{b1}Q_b + f_{b2}Q_b^2\right) + c_e P_{help,b}\right) \qquad \text{Eq. 6.3}$$

6.3.2.3   Engines

The running cost for diesel engines and gas engines running as CHP units is simply formed by their fuel consumption and the amount of electric energy that may be sold. This model is used in [Saether 1999].

Fuel consumption:

$$f_g = f_{g0} + f_{g1}P_g + f_{g2}P_g^2 \qquad \text{Eq. 6.4}$$

Running cost = fuel consumption - electricity sale:

$$c_g = T\left(f_g c_{f_g} - c_e P_g\right) = T\left(c_{f_g}\left(f_{g0} + f_{g1}P_g + f_{g2}P_g^2\right) - c_e P_g\right) \qquad \text{Eq. 6.5}$$

Assuming fix values on the electric efficiency and the total efficiency, the relation between the heat power and electric power produced may be expressed as:

$$P_g = \frac{1/\eta_{tot}}{1/\eta_e - 1/\eta_{tot}}Q_g \qquad \text{Eq. 6.6}$$

6.3.2.4   Back Pressure Plant

The running cost of the back pressure plant modelled depends on the fuel consumption and the amount of electric energy that may be sold, P_{net}. The amount of electric energy produced by the CHP depends on the supply temperature, i.e. increasing the supply temperature will result in a lower relative electricity production.

Back pressure line:

$$P = c_m Q_c \qquad \text{Eq. 6.7}$$

Heat power output:

$$P_0 = Q_c + \frac{P}{\eta_e} = Q_c\left(1 + \frac{c_m}{\eta_e}\right) \qquad \text{Eq. 6.8}$$

Fuel consumption:

$$f_c = \left(f_{c0} + f_{c1}P_0 + f_{c2}P_0^2\right) \qquad \text{Eq. 6.9}$$

Electricity usage:

$$P_{help,c} = e_{c0} + e_{c1}\frac{Q_c}{Q_{c\max}} + e_{c2}\left(\frac{Q_c}{Q_{c\max}}\right)^2 \qquad \text{Eq. 6.10}$$

Power to heat ratio:

$$c_m = \frac{\eta_e}{\eta - \eta_e} \qquad \text{Eq. 6.11}$$

Electric efficiency:

$$\eta_e = \frac{\alpha_e - T_{\sup}}{\beta_e} \qquad \text{Eq. 6.12}$$

Supply temperature:

$$T_{\sup} = \begin{cases} T_{\sup \max} - k_{\sup}\left(T_{out} - T_{\sup \dim}\right) & , T_{out} < T_{cut} \\ T_{\sup \min} & , T_{out} \geq T_{cut} \end{cases} \qquad \text{Eq. 6.13}$$

Running cost = fuel consumption – electricity sale:

$$c_c = T\left(f_c c_{f_c} - c_e P_{net}\right) = T\left(\left(f_{c0} + f_{c1}P_0 + f_{c2}P_0^2\right)c_{f_c} - c_e\left(c_m Q_c - P_{help,c}\right)\right) \qquad \text{Eq. 6.14}$$

6.3.2.5 Extraction Plant

Cool condensing mode:

$$P_{\max} = c_{m0}Q_0 + k_0 \qquad \text{Eq. 6.15}$$

Equi fuel line:

$$P = P_{\max} + c_v Q_H \qquad \text{Eq. 6.16}$$

Heat power output:

$$P_0 = \frac{P_{\max}}{\eta_e} + Q_0 = \left(\frac{1}{\eta_e} + \frac{1}{c_{m0}}\right)P_{\max} - \frac{k_0}{c_{m0}}$$
$$= \left(\frac{1}{\eta_e} + \frac{1}{c_{m0}}\right)P - c_v\left(\frac{1}{\eta_e} + \frac{1}{c_{m0}}\right)Q_H - \frac{k_0}{c_{m0}} \qquad \text{Eq. 6.17}$$

Fuel consumption:

$$f_e = f_{e0} + f_{e1}P_0 + f_{e2}P_0^2 \qquad \text{Eq. 6.18}$$

Electricity usage for heat production:

$$P_{help,heat} = e_{e0,heat} + e_{e1,heat}\frac{Q_H}{P_{0\max}} + e_{e2,heat}\left(\frac{Q_H}{P_{0\max}}\right)^2 \qquad \text{Eq. 6.19}$$

Electricity usage for electricity production, cool condense mode:

$$P_{help,el,cond} = e_{e0,el,cond} + e_{e1,el,cond}\frac{P_0}{P_{0\max}} + e_{e2,el,cond}\left(\frac{P_0}{P_{0\max}}\right)^2 \qquad \text{Eq. 6.20}$$

Electricity usage for electricity production, back pressure mode:

$$P_{help,el,bp} = e_{e0,el,bp} + e_{e1,el,bp}\frac{P_0}{P_{0\max}} + e_{e2,el,bp}\left(\frac{P_0}{P_{0\max}}\right)^2 \qquad \text{Eq. 6.21}$$

Electricity usage for electricity production:

$$P_{help,el} = (1 - \lambda_{bp})P_{help,el,cond} + \lambda_{bp}P_{help,el,bp} \qquad \text{Eq. 6.22}$$

where λ_{bp} is the back pressure coefficient; $\lambda_{bp}=1$ in back pressure mode and $\lambda_{bp}=0$ in cool condensing mode.

Running cost = fuel consumption – electricity sale:

$$c = T\left(f_e c_{f_e} - c_e P_{net}\right) = T\left(\left(f_{e0} + f_{e1}P_0 + f_{e2}P_0^2\right)c_f - c_e\left(P - P_{help,heat} - P_{help,el}\right)\right) \quad \text{Eq. 6.23}$$

6.3.2.6 Heat pump

The process efficiency and the condenser output of a heat pump both depend on the evaporation and the condensation temperature. In addition the efficiency and the output depend on the nominal capacity. This makes a heat pump a rather complicated heat production unit to model. The model described in this chapter was presented in [Nielsen 1996].

The carnot coefficient of performance, *CCOP*, describes the energy-exergy ratio for an ideal heat pump.

$$CCOP = \frac{T_{cond}}{T_{cond} - T_{evap}} \quad \text{Eq. 6.24}$$

where T_{cond} and T_{evap} are the temperatures of condensation and evaporation respectively. Introducing the carnot efficiency, η_{carnot}, describing the heat pump component size dependency as well as the compressor efficiency variation, the coefficient of performance for a real heat pump can be defined as:

$$COP = \eta_{carnot} CCOP \quad \text{Eq. 6.25}$$

In order to find the heat pump performance for a specific time step it is necessary to find the matching temperatures of evaporation and condensation. In the model described a number of simplifications are made to achieve a moderate complexity level of the heat pump model.

- The overall heat transfer coefficient of the condenser is constant. Thus the LMTD can be calculated knowing the condenser output.
- The LMTD of the evaporator is constant.
- The heat source cooling is constant.

The required model input information:

- Tabulated heat pump performance
- Condenser maximum output capacity and the LMTD of the condenser at a given state of reference.
- Return temperature of the distribution fluid as a function of a given outdoor temperature.
- The heat source temperature.

Since the overall heat transfer coefficient is assumed constant it can be calculated using equation Eq. 6.26:

$$UA_{cond} = \frac{P_{HP,ref}}{LMTD_{cond,ref}} \quad \text{Eq. 6.26}$$

When the overall heat transfer coefficient is known the LMTD at any other condenser output can be found from equation Eq. 6.27.

$$LMTD_{cond} = \frac{P_{HP}}{UA_{cond}} \quad \text{Eq. 6.27}$$

For evaporation and condensation at a constant temperature level, the temperatures of evaporation and condensation can be found from equation Eq. 6.28 and Eq. 6.29.

$$T_{evap} = \frac{T_{source,out} - T_{source,in}\, e^{\frac{T_{source,out} - T_{source,in}}{LMTD_{evap}}}}{1 - e^{\frac{T_{source,out} - T_{source,in}}{LMTD_{evap}}}}$$

Eq. 6.28

$$T_{cond} = \frac{T_{f,cond} - T_{ret}\, e^{\frac{T_{ret} - T_{f,cond}}{LMTD_{cond}}}}{1 - e^{\frac{T_{ret} - T_{f,cond}}{LMTD_{cond}}}}$$

Eq. 6.29

The condenser forward temperature which provides an equilibrium between the tabulated output capacity and the heating of the distribution fluid, $T_{f,cond}$-T_{ret}, is found by iteration. Thus the current states are known and the COP can be found from the carnot coefficient of performance and the tabulated carnot efficiencies.

If the condenser output capacity at the current system forward temperature is higher than the heat load, the condenser output must be decreased. At part load conditions the efficiency of the compressor must be added to the COP:

$$COP = \eta_{comp}\eta_{carnot}CCOP$$

Eq. 6.30

The cost function for the operation of the heat pump is given by equation Eq. 6.31.

$$c_{HP} = c_e \frac{P_{HP}}{COP}$$

Eq. 6.31

6.3.3 Model representation of hot water storage

The cost for the heat storage depends on the energy loss from the storage and the electricity cost for running pumps etc. When calculating the cost for energy loss the actual marginal cost for producing heat is used. This cost is calculated using the current marginal cost for each producing unit and the production distribution between the units.

Storage capacity:
$$Q_{s\,max} = \gamma_3 E_{s\,max}$$

Eq. 6.32

$$E_{s\,max} = \gamma_4 V$$

Eq. 6.33

Capacity limits:
$$-Q_{s\,max} \le Q_s \le Q_{s\,max}$$

Eq. 6.34

$$0 \le E_s \le E_{s\,max}$$

Eq. 6.35

The maximum charge and discharge rates are in Eq. 6.34 set symmetric. This assumption has been made for simplicity but may very well be set non symmetric.

Energy loss coefficient:
$$\gamma_1 = \gamma_5 V^{2/3}$$

Eq. 6.36

Running cost = energy loss to ambient + pump electricity cost:

$$c_s = T\left(c_{loss} + c_{pump}\right) = T\left(\gamma_1 c_s^* E_s + c_e \gamma_2 Q_s^2\right)$$

Eq. 6.37

where the marginal cost is calculated as:

$$c_s^* = \Delta c = \frac{\partial c}{\partial Q_1} \Delta Q_1 + \ldots + \frac{\partial c}{\partial Q_I} \Delta Q_I = [\Delta Q_1 \ldots \Delta Q_I] \nabla c$$

$$\Delta Q_i = \frac{Q_i}{\displaystyle\sum_{n=1}^{I} Q_n}, \quad i = 1, \ldots, I$$

Eq. 6.38

6.3.4   State at the end of the analysing period

As mentioned in chapter 4.1.1.3 all optimisation methods with no restrictions on the final state for the storage will end up having the storage empty at the end of the period. Three methods to regulate the final storage state were considered and the method utilized in this application is to extend the evaluation horizon.

In Figure 6-1 a resulting storage energy content is shown for three different evaluation horizons; 5, 10 and 15 days.
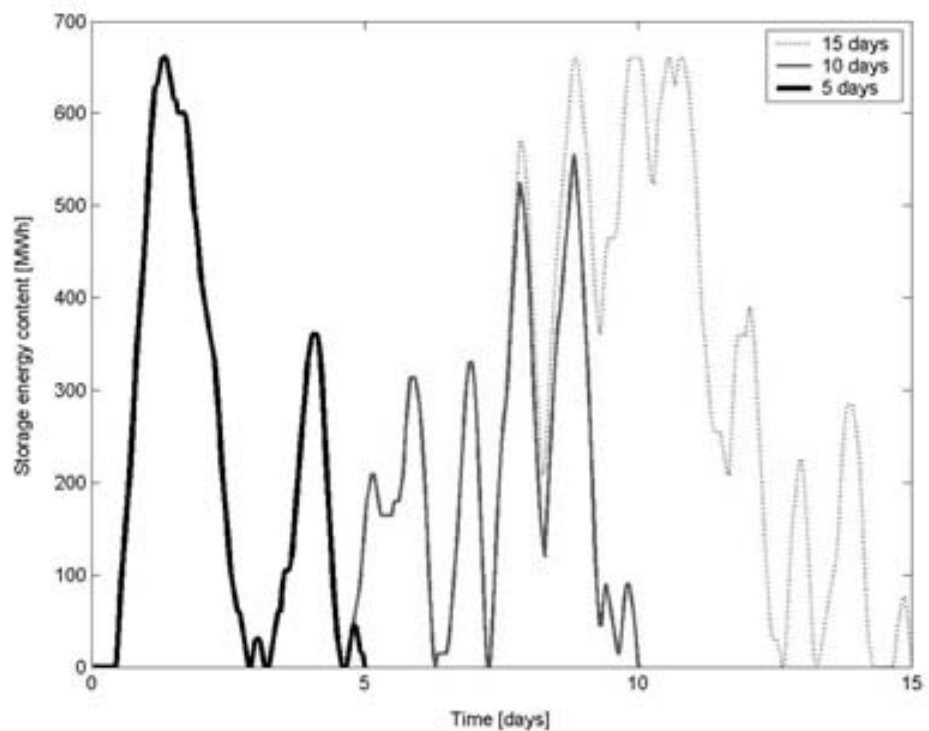

Figure 6-1 Extending the evaluation horizon.

The extension required to eliminate the influence of the final state depends on two factors. The first factor is the "true" amount of energy stored at the end of the evaluation period, i.e. the amount of energy that should be the case if the evaluation period was infinite. A large "true" amount of energy will require a longer extension of the evaluation period. This may be seen in Figure 6-1 where the 10-day case requires almost three days of extension whereas the 5-day case requires only a few hours.

The second factor affecting the extension period is the relation between the district heat load and the dispatch of the heat production units. If the district heat load, at the end of the evaluation period is such that the total production cost will gain more if the storage is being emptied at an earlier stage rather than at the end, this will also require a longer extension of the evaluation period.

The conclusion from the example illustrated in Figure 6-1 is that the evaluation horizon has to be extended at least three days in order to eliminate the influence from the final state.

6.4 Mathematical optimisation method

To find the optimal size of a thermal storage connected to a set of heat production units, a necessary prerequisite is the optimal operation of these units i.e. solving the unit commitment and the load dispatch problem. The thermal storage will here be treated as a heat production unit, however somewhat different, and hence a part of the unit commitment problem.

6.4.1 The overall optimisation method

The optimal volume, the volume that minimises the production cost, is found using a standard interval elimination search routine. For each storage size V evaluated during the search, the optimal production cost for the complete evaluation period, $c_{opt}(V)$, is found using non-linear programming for solving the load dispatch problem and dynamic programming for solving the unit commitment problem.

The algorithm for finding the optimal storage size is discribed below.

1. Choose an initial storage size V_0.
2. Solve the load dispatch problem for all feasible storage operations,states, and for all time steps.
3. Solve the unit commitment problem. This will give the optimal operation of the plant and the optimal operation of the storage resulting in an optimal cost $c_{opt}(V_0)$
4. Find a new storage size, V_m and jump to step 2. Stop the search when the difference between $c_{opt}(V_m)$ and $c_{opt}(V_{m-1})$ is sufficiently small.

As an alternative to the search procedure described above it may be illustrative to run the optimization on an interval of store volumes.

6.4.2 Solving the Load Dispatch Problem

In order to solve the load dispatch problem for each step in time, find the optimal production unit operation, given a number of active production units and given the heat storage power.

Let $Q=[Q_1,...,Q_I]^T$ denote the heat production of the given production units and Q_s the given heat storage power. The problem is then to find the combination $Q=[Q_1,...,Q_I]^T$ that fulfills the load requirement

$$Q_s + \sum_{i=1}^{I} Q_i = Q_d \qquad \text{Eq. 6.39}$$

but also minimises the total production cost, i.e. to solve:

$$\min_Q \quad c = f(Q) = f(Q_1,...,Q_I) = c_s + \sum_{i=1}^{I} c_i(Q_i)$$

$$\text{Eq. 6.40}$$

$$subject\ to \quad g(Q) = \sum_{i=1}^{I} Q_i + Q_s - Q_d = 0$$

Note that the cost function depends on which production units are active. E.g. if a boiler and a back pressure CHP unit as represented in chapter 6.3.2 are both active, we get the following cost function:

$$c = T\left(\gamma_1 c_s^* E_s + c_e \gamma_2 Q_s^2 + c_{f_b}\left(f_{b0} + f_{b1}Q_1 + f_{b2}Q_1^2\right) + c_e p_{help,b} + ...\right.$$

$$\left. ... + \left(f_{c0} + f_{c1}Q_2\left(1+\frac{c_m}{\eta_e}\right) + f_{c2}\left(\left(1+\frac{c_m}{\eta_e}\right)Q_2\right)^2\right)c_{f_c} - c_e\left(c_m Q_2 - p_{help,c}\right)\right) \qquad \begin{matrix}\text{Eq.}\\\text{6.41}\end{matrix}$$

where $Q_1=Q_b$ and $Q_2=Q_c$.

The problem is solved using the Lagrange Multiplier Method. This method is based on the fact that the gradient of the objective function f and the gradient of the constraint g have to be parallell in a local optimum.

$$\nabla f(Q_1,...,Q_I) = \lambda \nabla g(Q_1,...,Q_I)$$

<div align="right">Eq. 6.42</div>

Together with the constraint $g(Q)=0$, we have $I+1$ unknowns and $I+1$ equations. Solving these equations will thus give the combination Q that minimises the production cost.

A drawback with the Lagrange Multiplier Method is that it can only handle equality contraints, i.e. unequalities describing heat production capacity limits have to be considered separately. A method to guarantee a proper operational range for the production units is presented in [Nilsen 1994] and is resumed below:

1. If the Lagrange Multiplier solution contains one unit running above its rated capacity the unit is fixed at its rated maximum. If several units are running above their rated capacity, the one with the lowest marginal cost is set to maximum.
2. If the Lagrange Multiplier solution contains one unit running below its minimum rated capacity the unit is fixed at its rated minimum. If several units are running below their minimum rated capacity, the one with the highest marginal cost is set to minimum.
3. The fixed units and their respective heat load are removed and the problem is solved again.

The procedure described above is valid when using units producing heat only or CHP units where the relation between the heat power and the electric power produced is constant. If production units are used where the electric power is independent of the heat power, an additional problem constraint regarding the electric power demand must be added:

$$g(P) = \sum_{i=1}^{I} P_i - P_d = 0$$

<div align="right">Eq. 6.43</div>

6.4.3   Solving the Unit Commitment Problem

The original problem of unit commitment is to decide which production units should be engaged during a certain time period. Here, the heat storage is added and hence the unit commitment problem is not only to find which production units should be active but also to find the optimal storage energy level for each time step. The optimal production cost for all these combinations and time steps is already calculated (load dispatch).

To indicate which production units are active and how the storage tank is operated, a system state is defined:

$$x_k = x_k(u, E_s), \quad k = 1,...,K$$

<div align="right">Eq. 6.44</div>

where u denotes the heat production units active, E_s the storage energy content and K the number of states i.e. the state contains information about which units are producing heat and how much energy is stored in the tank.

The storage energy content, E_s, is discretised in order to get a finite number of states:

$$E_s \in \{0, \Delta E_s, 2\Delta E_s,..., E_{s\,max}\}$$

<div align="right">Eq. 6.45</div>

which gives the following expression for the total number of states:

$$K = \left(\frac{E_{s\,max}}{\Delta E_s} + 1\right)(N^2 - 1)$$

<div align="right">Eq. 6.46</div>

where N indicates the number of available production units.

A state transition cost $f(x_k(n),x_k(n-1))$ is associated with each time step n which is the sum of the production cost for the former state and possible start and stop costs if the combination of active production units has been changed during the time step. Solving the unit commitment problem is to find the state path through the time interval evaluated that minimises the accumulated state transition cost c_{dp}. The procedure is illustrated in Figure 6-2.
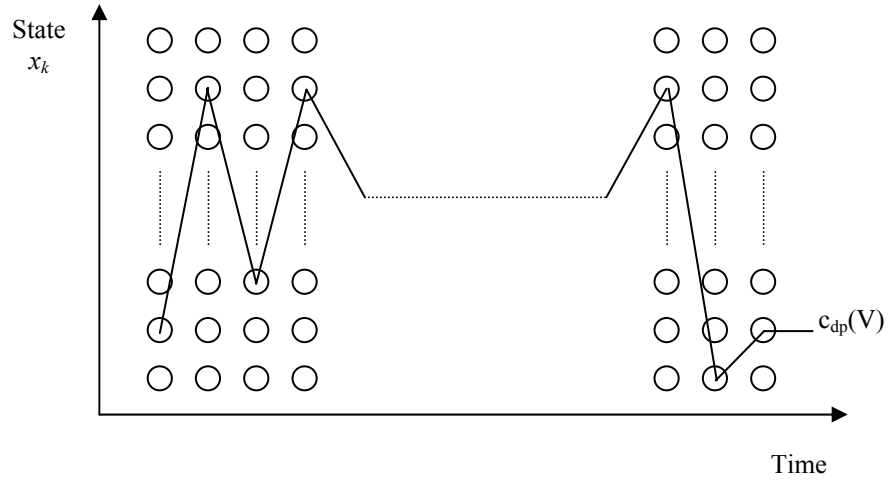

Figure 6-2. Solving the unit commitment problem finds the path that minimises the accumulated state transition cost.

Dynamic programming is used for solving this problem. Define the accumulated transition cost function as:

$$c_{dp}(x_k(n)) = c_{dp}(x_k(n-1)) + f(x_k(n), x_k(n-1))$$

Eq. 6.47

The technique in dynamic programming is to, for each time step and for all feasible states, find the smallest accumulated transition cost:

$$c_{dp}^*(x_k(n)) = \min_{x_k(n-1)} c_{dp}(x_k(n-1)) + f(x_k(n-1), x_k(n-1))$$

Eq. 6.48

and then beginning at the end of the time period, back trace the optimal path. When the optimal state path is found the minimum production cost for the current size of the store c_{opt} is found and equals the accumulated transition cost in the last time step c_{dp}^*.

For a given time step n, not all states are feasible. The first condition is that the load requirement is fulfilled by the maximum capacity of the heat producing units together with the heat storage discharge:

$$\sum_i Q_{i\,max} + Q_s \geq Q_d$$

Eq. 6.49

The second condition is that Q_s does not exceed the limits for charging and discharging the storage:

$$-Q_{s\,max} \leq Q_s \leq Q_{s\,max}$$

Eq. 6.50

Since the load dispatch problem only needs to be solved for feasible states, discarding unfeasible states will lower the total CPU time.

6.4.4   An optimisation algorithm

Since the idea is to find the storage volume that enables the lowest possible running cost the main work will be undertaken in calculating optimal heat production costs. This is done by identifying all possible system states, calculating the state transition cost for all time steps and then choosing the state path over time which gives the lowest production cost.

This will give production cost as a function of storage volume and the optimal storage volume may then be found using a simple search routine.

In order to solve the load dispatch problem we only need the storage heat power and the production units active i.e. it is not needed to know the storage energy content. Start the procedure with solving the load dispatch problem for all feasible levels of heat storage power, all combinations of production units and for all time steps. This will give the optimal production cost and the optimal marginal cost as a function of storage heat power and the production units active.

The next step is to calculate the state transition costs needed for solving the unit commitment problem. Since a transition between two states implies a jump between two storage energy levels i.e. storage heat power, and possibly the start or stop of a number of production units, the transition cost may be given through interpolation among the previously calculated production costs and adding the cost for starting or stopping production units.

The purpose of this somewhat awkward method for calculating the transition costs is to avoid solving the load dispatch problem for all state transitions, i.e. transitions between storage energy levels, since the number of possible storage energy level transitions is essentially larger than the number of feasible levels of storage heat power.

Finally the storage heat loss cost is added to the transition costs using the marginal cost already derived when solving the load dispatch problem.

When all the transition costs are fully derived, the unit commitment problem may be solved which finally gives the optimal production cost.

Storing all the transition costs is a rather memory consuming procedure and it may be necessary to split the unit commitment analysis into a series of executions.

The procedure described above may be summarised using the following algorithm:

```
Initialise all model parameters
Read input signals (electricity price, outdoor temperature, heat load)
while          not convergence in the storage volume search routine

               choose new heat storage volume
               solve the load dispatch problem c_ld=c_ld(Q_s,u,t)
               calculate all transition costs f(x_n,x_n-1)=interpolate(c_ld)
               add start/stop costs and heat storage loss cost
               solve the unit commitment problem
end
```

6.5   Model input

In order to run the optimisation model a number of input signals are needed. These signals are:

- Outdoor temperature, T_{out}
- District heat load, Q_d
- Electricity spot market price, c_e

If the district heat load is not available it may instead be modelled e.g. as a linear function of the outdoor temperature with a base load for DHW.

6.6   Investment Cost

To evaluate the total heat storage cost it is necessary to consider the investment cost. A fixed annual installment is used to calculate the yearly investment cost and the heat storage purchase price as a function of size is shown in Figure 6-3.
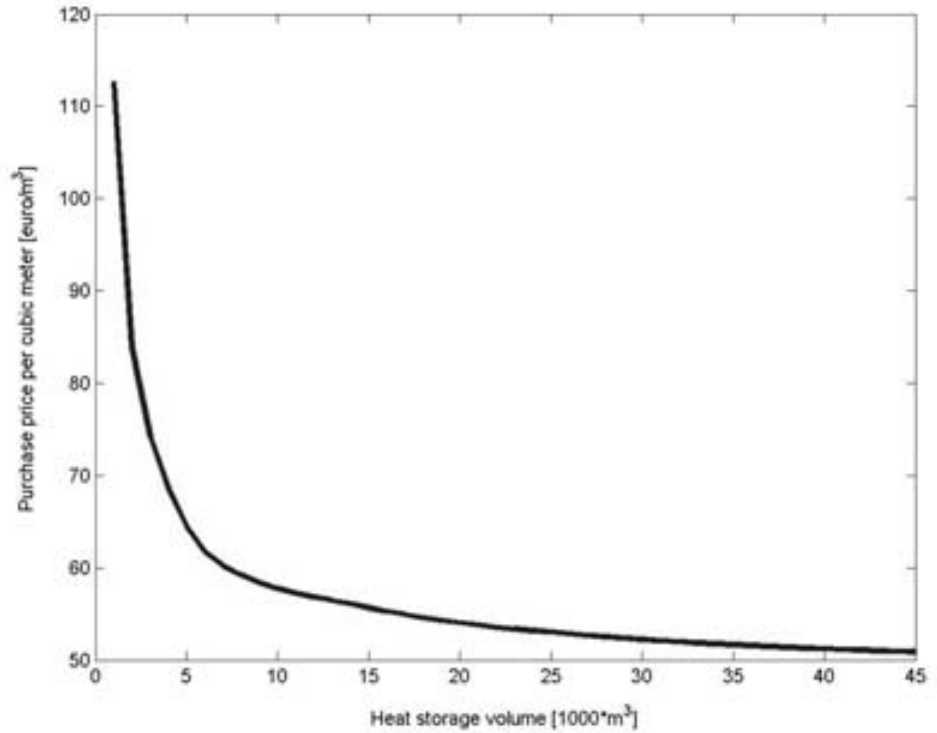
Figure 6-3. The specific investment cost as a function of storage volume. These figures stems from the purchase prices listed in [Nuutila] but have been corrected for inflation.

The annual cost due to investment is calculated as:

$$c_i = I \frac{i}{1-(1+i)^{-N}}$$

Eq. 6.51

where I is the purchase price, i is the discount rate and N is the number of instalment payment periods.

6.7 Numerical Example

In this section the optimisation methodology described will be applied to a plant consisting of a back pressure CHP unit, a heat boiler and a heat store. The chosen model parameters are presented in **Table 6-1**.

Table 6-1. Constructed optimisation parameters.

Heat store parameters					
ΔE_s	ΔQ_s	γ_2	γ_3	γ_4	γ_5
15e6	15e6	2e-11	0.15	44	2.4e-7
Heat boiler parameters							
$Q_{b\max}$	f_{b0}	f_{b1}	f_{b2}	e_{b0}	e_{b1}	e_{b2}	c_{f_b}
100e6	74289	1.0518	3.6524e-10	0.73125e6	0.175e6	0.09375e6	28.35e-6
Back pressure CHP unit parameters							
$Q_{c\max}$	f_{c0}	f_{c1}	f_{c2}	e_{c0}	e_{c1}	e_{c2}	c_{f_c}
182e6	6773	1.052342	8.17056e-11	1.25e6	1.303296e6	1.582418e6	13.08e-6
α_e	β_e	$T_{\sup\min}$	$T_{\sup\max}$	$T_{\sup\dim}$	T_{cut}	$k_{\sup}$	
185	500	82	130	-20	8	1.714	

The length of the evaluation period is one year. The electricity price used in the optimisation is the nordic market for electric energy exchange, Nordpol, spot price during year 2002 and the district

heat load is given by the historical data for 2002 supplied by a district heat company. During this year, 712 000 MWh were delivered to the district.

Figure 6-4 displays the production cost and the total cost, i.e including investment, versus the storage size. The investment cost is added as described in chapter 6.6 where the interest rate is 5% and the instalment period is set to 25 years The analysis results showed that optimal storage size is approximately 27 000 m^3 and the annual savings are 3 %.
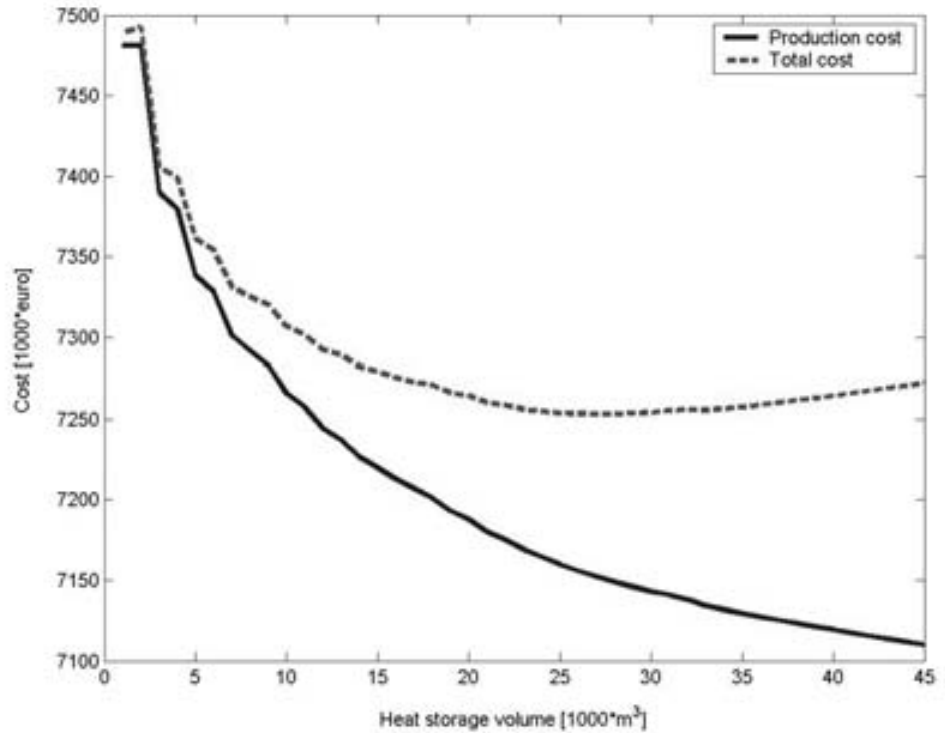
Figure 6-4. Optimal running cost as a function of storage volume.

In this numerical example the heat store is utilized in two ways. The first reason for using the store is to avoid running the expensive heat boiler. The second reason for using the storage is to enable the CHP to produce more heat than required by the district heat load, which is economically profitable during periods of high electricity prices. The electricity price for a 24 hour period is shown in Figure 6-5 and in Figure 6-6 it is shown how the heat storage power and the heat storage energy content varies during the same period. In this example it is clearly seen that the store is charged during the day when the electricity price is high and discharged during the night.
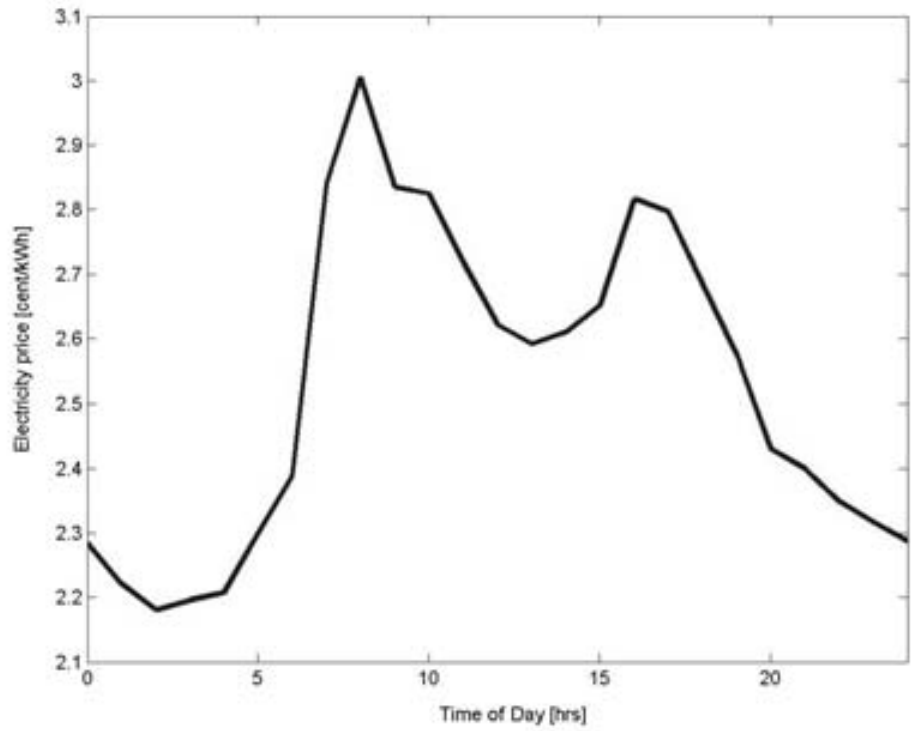
Figure 6-5. Electricity price during a 24 hour period in January starting at 00:00.
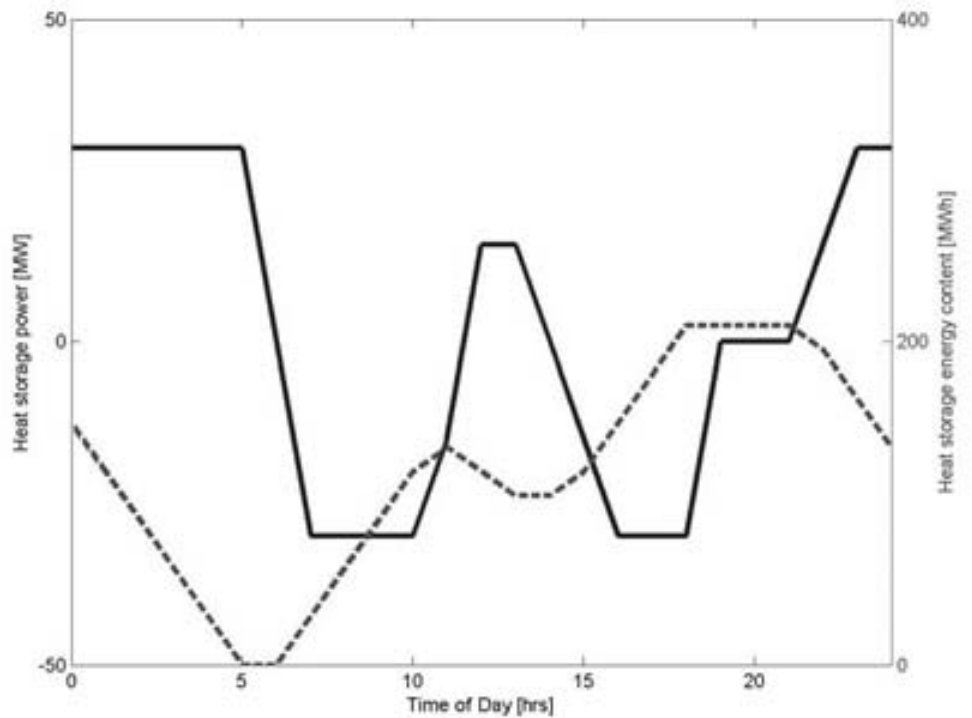

Figure 6-6. Heat storage power and heat storage energy content during a 24 hour period in January starting at 00:00. The storage is charged during the morning and afternoon electricity price peak.

6.8   Uncertainties

In chapter 4.4 uncertainties in input data and methodology uncertainties were discussed and in this chapter some of these uncertainties are studied using the optimisation methodology presented.

6.8.1   *Future heat load uncertainty*

In Figure 6-7 it is shown how the optimal storage volume is related to the amount of peak load required to satisfy the district heat load. The optimisation parameters used are the same as in chapter 6.7 except that the maximum capacity of the heat water boiler is increased from 100 MW to 200 MW and the evaluation period is reduced from a whole year to January to March 2002. The district heat load is linearly scaled in order to vary the amount of peak load required.
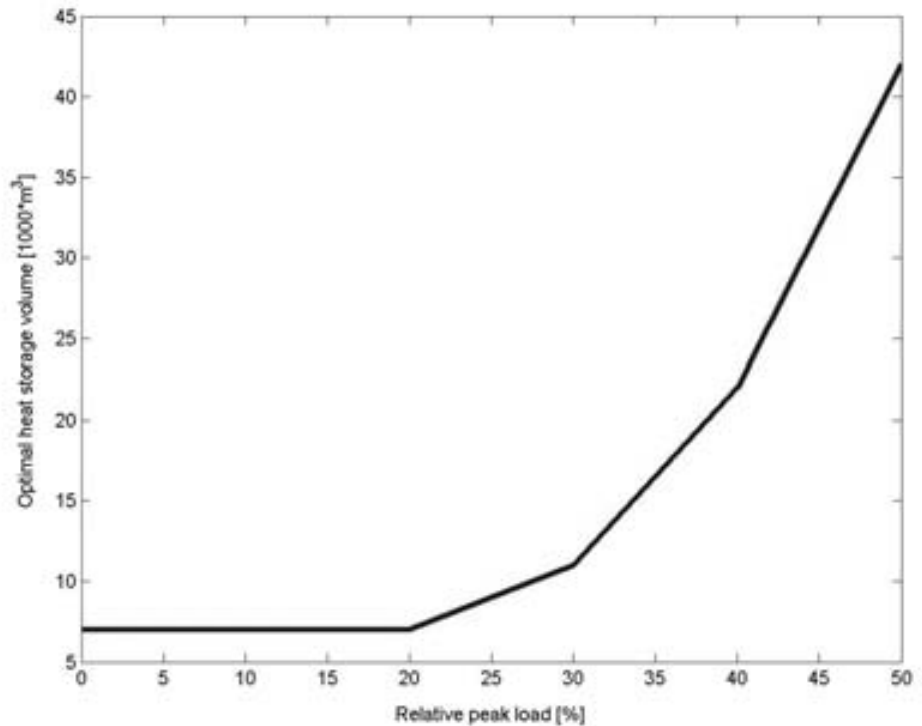

Figure 6-7. The optimal heat storage volume is strongly connected to the relation between the maximum capacity of an inexpencive base load unit and the maximum district heat load.

It is seen that in this example the amount of storage needed is strongly connected to the amount of peak load required.

6.8.2   *Discretisation uncertainty*

In **Table 6-2** it is shown how the optimal storage volume is affected by the discretisation level of the storage. The optimisation parameters used are the same as in chapter 6.7.

Table 6-2. Discretisation uncertainty

Storage energy content discretisation [MWh]	15	20	25	30
Optimal storage volume [1000*m^3]	27	25	25	26

This example indicates that the degree of discretisation in the storage energy level does not significantly affect the optimal storage volume.

7 Summary and conclusions

This report presents a methodology to assist the planning of thermal storage into a DH&C plant. The methodology may be divided into the solving of two problems; existence and dimensioning. Further analysis on the shape and technical design of the store is presented.

Minimising the heat loss to the surroundings of a cylinder shaped store implies an H/D ratio equal to 1,0. However, if the most important issue is to minimise the amount of non-productive volume the best H/D ratio is 2,0. However some additional height is inevitable due to the space required for diffusors and also for steam pockets, if present. This is quite consistent with the existing stores in the Nordic countries where a majority of the stores have a H/D ratio in the range of $1,0 – 2,0$.

A simplified method for calculating the economic benefits by using heat storage in different district heating systems is presented and demonstrated. The method is based on historical data. The calculations for some cases show that the simplified method is applicable as a first approach for an investment decision in heat storage.

In the case study we find that there are 42 days where the heat load in some hours is more than 5 MW below the limit on 65 MW. During these hours it would be possible to produce to the storage. With storage of 3000 m^3 the extended use of the base load for these hours is 2.4 GWh.

A methodology for solving the storage dimensioning problem is presented together with a numerical example with a district heat load from a real DH system and where the heat is produced by a back pressure CHP in combination with a oil fired heat boiler. For an annual heat load of 712 000 MWh the optimal heat storage volume is approximately 27 000 m^3 and the savings are about 3 %.

Operational optimisation models found in relevant publications are used for finding the optimal operatation of a given DH&C plant together with a heat store of a certain size such that the total operating cost is minimised. All models are second order polynomials and the operational optimisation is conducted using the lagrange multiplier method. The operational optimisation is done for each step in time and for all combinations of heat producing units active and for all possible levels of charging/discharging the storage. This problem is called the dispatch problem. Once this problem is solved for all combinations and for all time steps the next stage is to find the series of combinations through the time interval evaluated that minimises the total cost. This problem is called the unit commitment problem, i.e. how to optimally employ the heat production units, and the solution is found using dynamic programming.

The procedure described above is performed for each storage volume considered and results in a optimum total operational cost. To find the optimum storage volume one may either run a search routine or simply run the optimisation procedure for a number of volumes within an appropriate interval. The latter is advisable since the optimum of this type of problem has shown to be very flat.

Both the simplified and the optimisation method give approximately the same size of the storage and the same economic result, with a slight overestimation of the storage size in the simplified method. This is probably due to the fact that losses from the store are not included in the simplified method.

References

References in the report.

[Benonysson 1991]   Benonysson A., *Dynamic Modelling and Operational Optimization of District Heating Systems* , PhD Thesis, Technical University of Denmark, Lyngby, Denmark 1991.

[Dahl 1993]   Dahl, Jan, *Short Term Water Heat Storage, Studies of velocity and temperature fields and their importance for sizing the storage,* Doctoral thesis from Luleå University of Technology 1993:131D, ISSN 0304-8373.

[Dincer et. al. 2001]   Dincer, I., Rosen M. A. (2001); *Thermal Energy Storage*, John Wiley & sons ltd, 2001, ISBN 0-471-49573-5.

[Dotzauer 1997]   Dotzauer E., *Algorithms for Short-Term Production-Planning of Cogeneration Plants* , Tek Lic Thesis, Linköping University, Linköping, Sweden 1996.

[Dotzauer 2001]   Dotzauer E., *Energy System Operation by Lagrangian Relaxation*, Thesis No 665, ISBN 91-7219-900-8, Department of Mathematics, Linköping Univeristy, Sweden, 2001

[Eriksson et al 1989]   Eriksson H., Gummerus P., Leckner B., *Optimal Operation of Heat Storage in District Heating Systems.* International Symposium on District Heat Simulation, Nordic Council of Ministers Energy Research Cooperation, 13-16 April, Reykjavik, 1989.

[Eriksson 1994]   Eriksson H., *Short Term Operation of District HeatingSystems.* PhD Thesis, Department of Energy Conversion, Chalmers University of Technology, Gothenburg, Sweden 1994.

[Fredriksen Werner 1993]   Fredriksen S., Werner S., *Fjärrvärme – Teori, teknik och funktion.* (in swedish), ISBN 91-44-38011-9, Studentlitteratur AB Lund, Sweden, 1993

[Goodfellow 2001]   Goodfellow, H. , Tahti, E. (2001); Industrial ventilation, design guidbook, ISBN 0-12-289676-9.

[Hermansson 1993]   *Short Term Water Heat Storage*, Doctoral thesis from Luleå Univesity of Technology 1993:127D, ISSN 0348-8373.

[NE 1980]   *Hetvattenteknik, resultat, utvecklingsläge och förutsättningar. Resultatrapport NE 1980:6* (in swedish)

[Nielsen 1996]   *A Model for optimization and Analysis of Energy Flexible Boiler Plants for Building Heating purposes*, PhD Thesis, 1996:39, Norwegian Institute of Technology, Trondheim, Norway, 1996.

[Nilsen 1994]   Nilsen J. O., *A Computer Model for Planning of Energy Systems with time dependent components and boundary conditions*, PhD Thesis, Norwegian Institute of Technology, Trondheim, Norway 1994.

[Nordvarme 1993]   Petersen, Erling; Hahn, Ragnar; Lappalainen, Juha; *Korttidslagring af varmt vand i tanke over jorden*, Danske Fjernvarmeværkers Forening, bog nr. 912.

[Nuutila]   Nuutila M., *Large thermal storage systems in district heating and CHP systems*, Finnish district heating association, Finland.

[Ravn, Rygaard 1993] Ravn H. F., Rygaard J. M. *Optimal Scheduling of Coproduction with a Storage*, Technical Report IMSOR, Technical University of Denmark, Lyngby,Denmark, 1993

[Saether 1999] Saether S., *Thermal Heat and Power Production with Models for Local and Regional Energy Systems*, PhD Thesis, The Norwegian University of Science and Technology, Trondheim,Norway, 1999

[Sundberg 2003] Sundberg V., *Hot Water Storage Tanks*, Work report, EPT2004:005, The Norwegian University of Science and Technology, Department of Energy and Process Technology, Trondheim,Norway, June 2003

[Wigbels et al 2002] Wigbels M., Althaus W., Lucht M., *Nonlinear Optimization in CHP Applications,* International Symposium on District Heating and Cooling, Nordic Energy Research, 14-16 August, Trondheim, Norway, 2002

[Werner 1997] Werner S., *Dynamiska värmelaster från fiktiva värmebehov,* (in swedish), Svensk Fjärrvärme, FOU 1997:10

[Zhao et al 1998] Zhao H., Holst J., Arvastson L., *Optimal Operation of Coproduction with a Storage,* Energy Vol 23, No 10/1998, pp 859-866
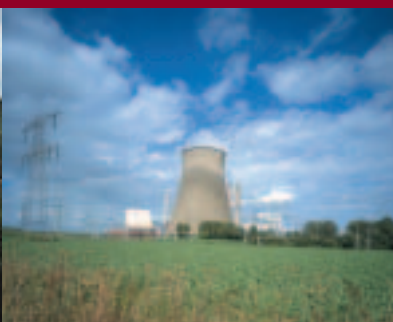
IEA DHC|CHP

International Energy Agency
IEA Implementing Agreement on District Heating and Cooling,
including the integration of CHP

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